Orthogonality Relations for a Stationary Flow of Ideal Fluid

(27 July 2018)

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For a real solution

$$u_i, p \in \mathcal{S}(\mathbb{R}^n) \ (i = 1, \dots, n)$$

to the Euler stationary equations for ideal fluid, the speaker and his research group derive an infinite series of identities (orthogonality relations) that equate some linear combinations of *m*th degree integral momenta of the functions $u_i u_j$ and p to zero (m = 0, 1, ...). In particular, zeroth degree orthogonality relations are:

$$(u_i, u_j)_{L^2(\mathbb{R}^n)} = 0 \ (i \neq j) \text{ and } \|u_i\|^2_{L^2(\mathbb{R}^n)} = -\int_{\mathbb{R}^n} p\,dx \ (i = 1, \dots, n) .$$

Orthogonality relations of degree m are valid for a solution

$$u_i \in W^1_{2,m}(\mathbb{R}^n), p \in W^1_{1,m}(\mathbb{R}^n) \text{ , where } W^1_{k,m}(\mathbb{R}^n)$$

is the weighted Sobolev space with the norm

$$\|u\|_{W^1_{k,m}(\mathbb{R}^n)} = \|(1 + |x|)^m u\|_{W^1_k(\mathbb{R}^n)} .$$