The Inverse Back-scattering Problem

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An acoustic medium is probed by plane waves from all directions and the medium response is measured back in the same directions. The goal is the recovery of the acoustic properties of the medium from this back-scattered data. Specifically, suppose $q(x)$ is a compactly supported smooth function on $\mathbb{R}^3$, representing the acoustic property of a medium. For each unit direction ω in $\mathbb{R}^3$, let $u(x, t; \omega)$ be the solution of the initial value problem

$$u_{tt} - \Delta_x u + q(x)u = 0, \qquad (x,t) \in \mathbb{R}^3 \times \mathbb{R}$$
$$u(x, t; \omega) = \delta(t - x \cdot \omega), \qquad x \in \mathbb{R}^3, \; t << 0.$$

The back-scattering data, in the direction ω, with delay s, is

$$\beta(s, \omega) = \lim_{r \to \infty} u(r\,\omega, r - s\,\omega), \qquad s \in \mathbb{R}, \; \omega \in \mathbb{R}^3 \; |\omega| = 1.$$

The inverse back-scattering problem is the study of the non-linear map

$$F: q(\cdot) \to \beta(\cdot, \cdot),$$

particularly the injectivity and the inversion of F. The speaker and his group survey the results for this long-standing unsolved problem, based on work done with Gunther Uhlmann.