

The Local Inverse Problem for the Geodesic X-ray Transform on Tensors and Boundary Rigidity

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András Vasy

Stanford University

In this talk, based on joint work with Plamen Stefanov and Gunther Uhlmann, the speaker will discuss the geodesic X-ray transform on a Riemannian manifold with boundary. The geodesic X-ray transform on functions associates to a function its integral along geodesic curves, so for instance in domains in Euclidean space along straight lines. The X-ray transform on symmetric tensors is similar, but one integrates the tensor contracted with the tangent vector of the geodesics. The speaker will explain how, under a convexity assumption on the boundary, one can invert the local geodesic X-ray transform on functions, i.e. determine the function from its X-ray transform, in a stable manner. The speaker will also explain how the analogous result can be achieved on one forms and 2-tensors up to the natural obstacle, namely potential tensors (forms which are differentials of functions, respectively tensors which are symmetric gradients of one-forms).

Here the local transform means that one would like to recover a function (or tensor) in a suitable neighborhood of a point on the boundary of the manifold given its integral along geodesic segments that stay in this neighborhood (i.e. with both endpoints on the boundary of the manifold). The speaker and his collaborators' method relies on microlocal analysis, in a form that was introduced by Melrose.

The speaker will then also explain how, under the assumption of the existence of a strictly convex family of hypersurfaces foliating the manifold, this gives immediately the solution of the global inverse problem by a stable 'layer stripping' type construction. Finally, the speaker will discuss the relationship with, and implications for, the boundary rigidity problem, i.e. determining a Riemannian metric from the restriction of its distance function to the boundary.