Study of surface plasmon polaritons (SPPs) from periodic metallic arrays by coupled mode theory

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Acknowledgement
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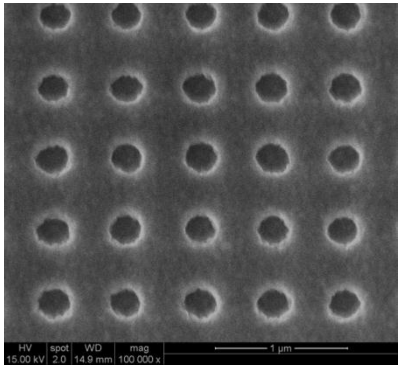

Outline

- How we fabricate and characterize periodic arrays
- Coupled mode theory (CMT)
- Determination of the absorption and radiative decay rates of SPPs
- Maximize the field strength by matching the absorption and radiative decay rates
- Control the phase difference between p- and s-polarizations for SPR sensing

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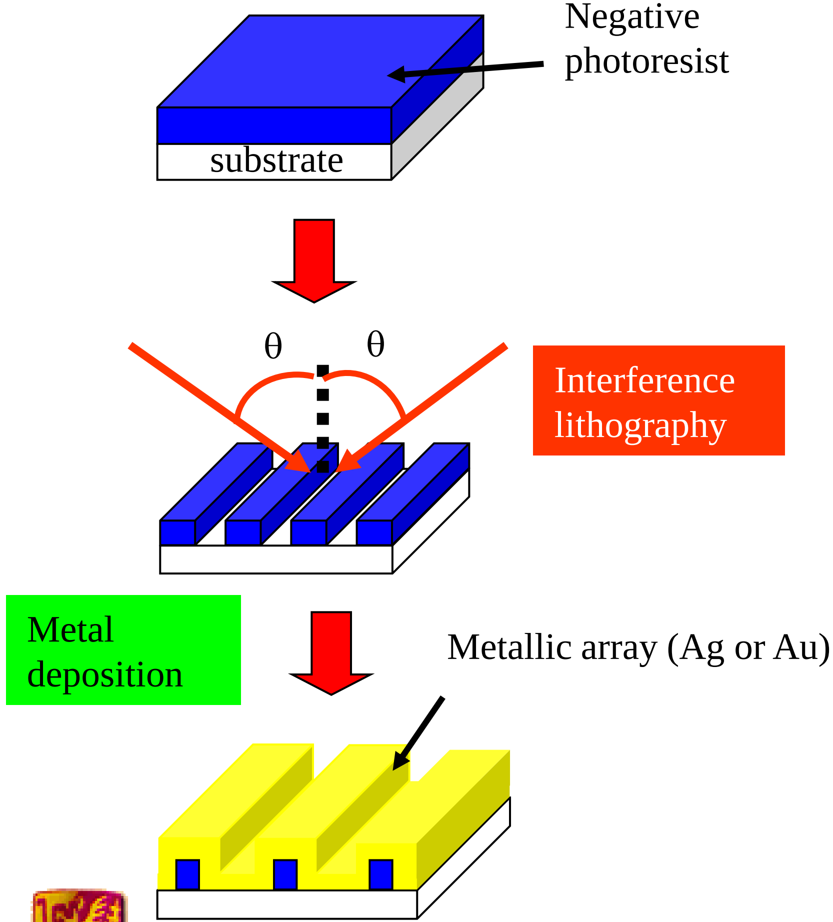
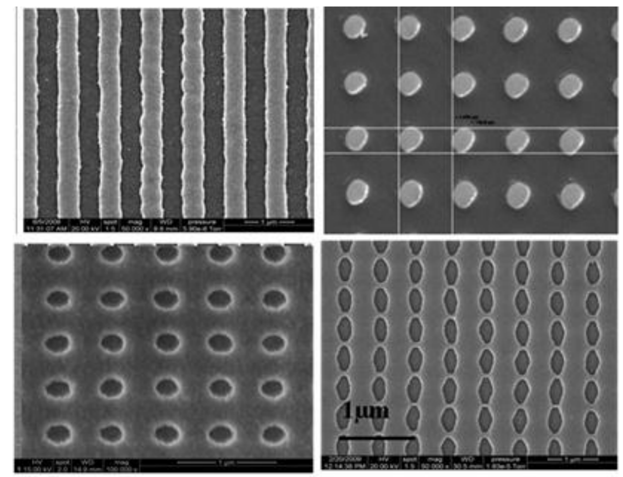
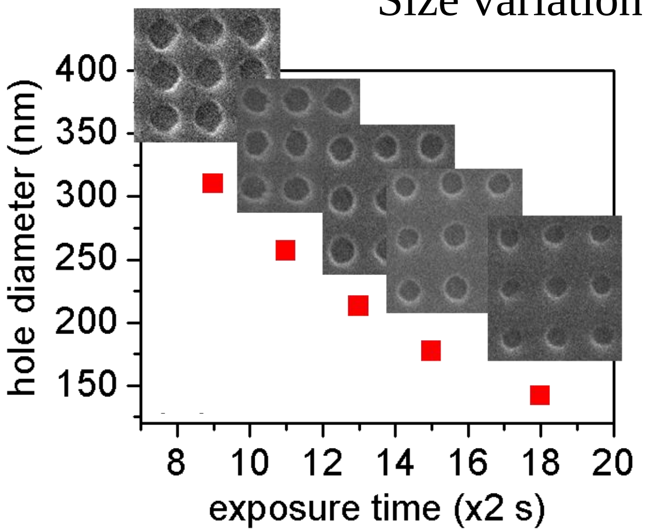
Fabrication of periodic metallic hole arrays

Combine interference lithography and thin film deposition





Size variation ±10%



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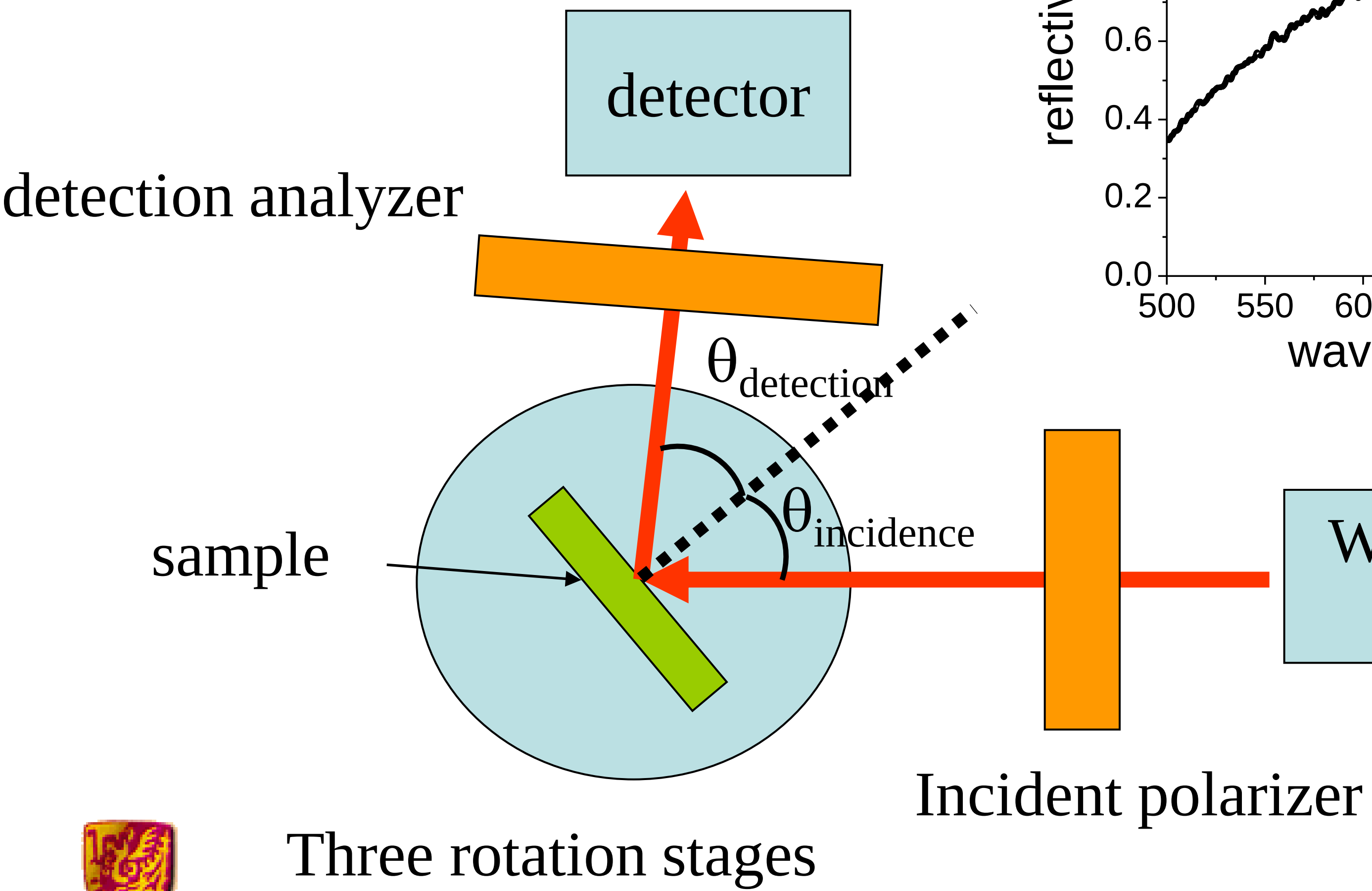
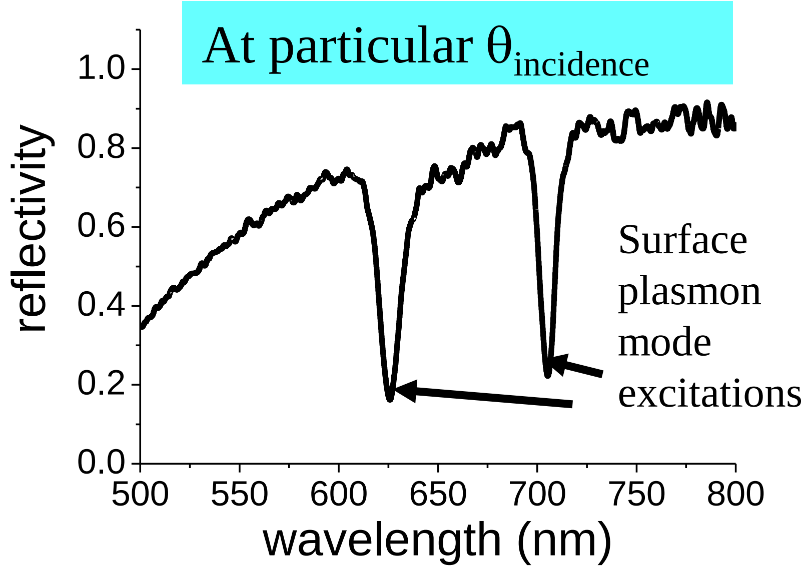
J. Li et al, APL 92, 213106 (2008); J. Li et al, APL 94 033101 (2009); J. Li et al, APL 94, 183112 (2009); K.C. Hui et al, APL 95, 063110 (2009)

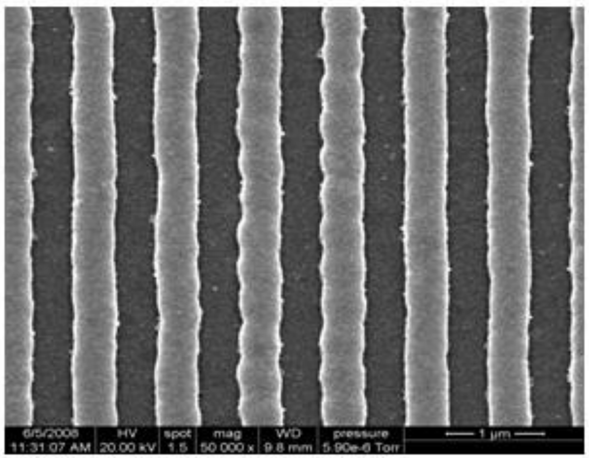
Optical characterization

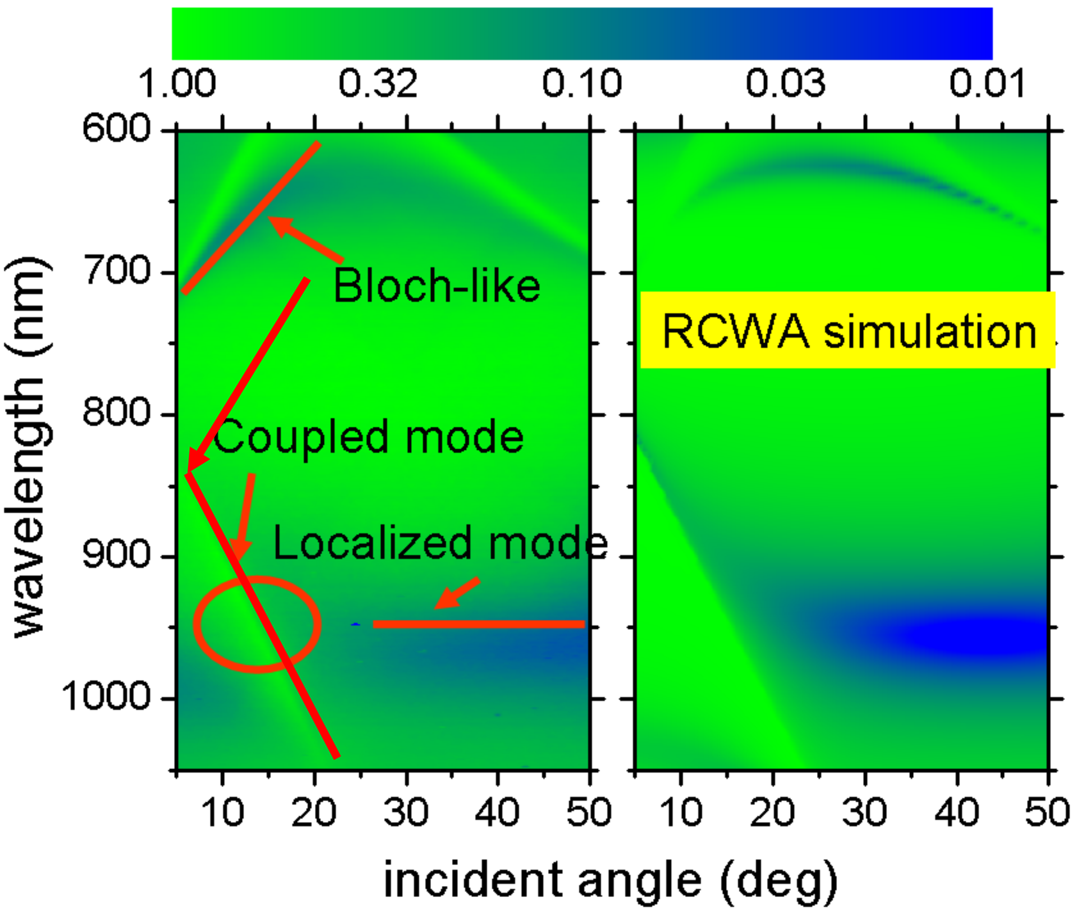
Measure specular or total reflectivity





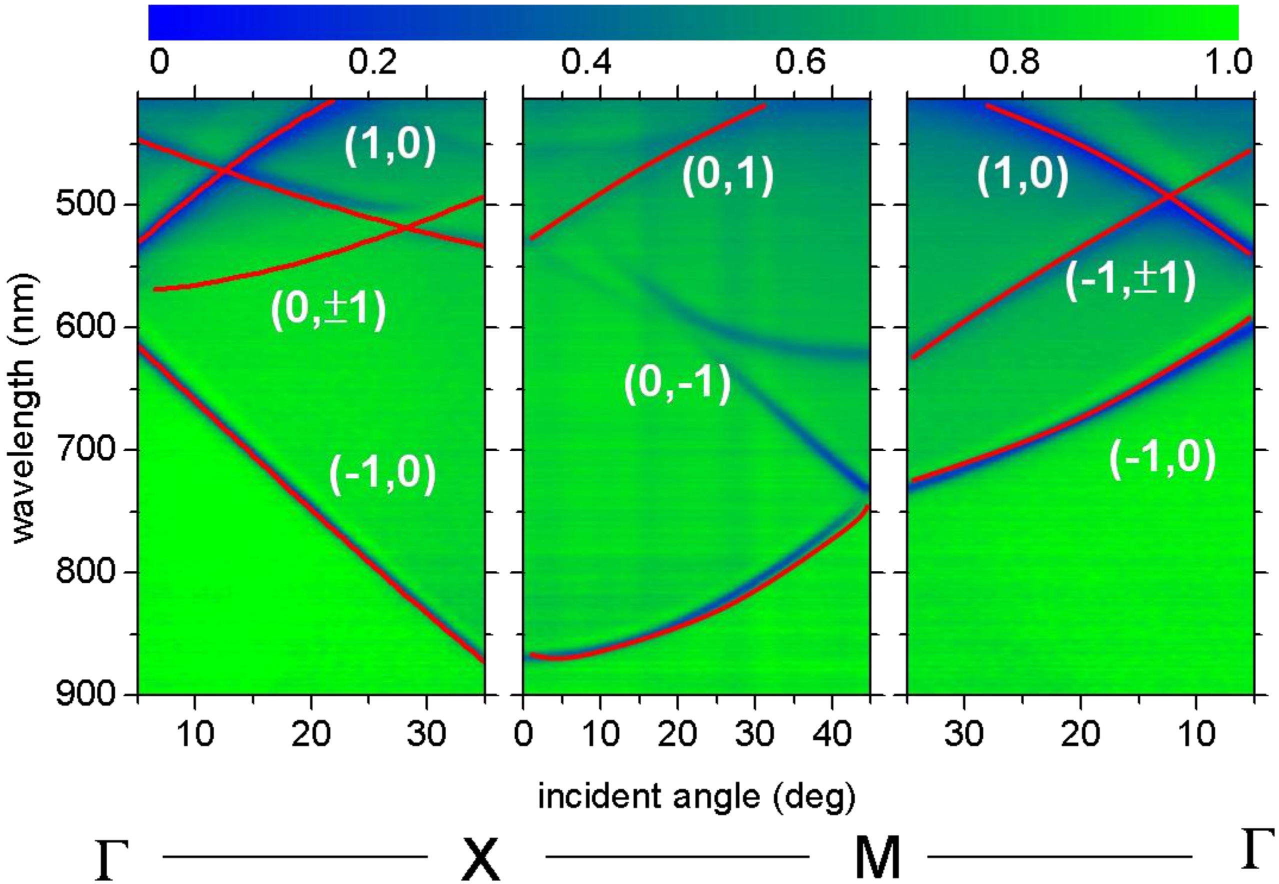
P-reflectivity mapping from 1D Au grating

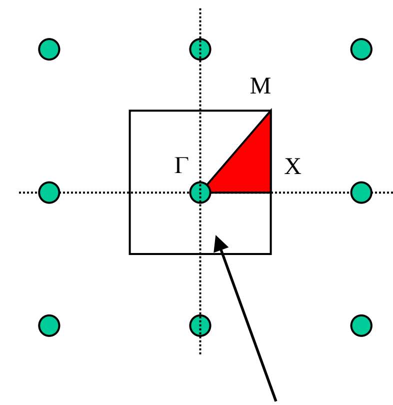




Blue region : low reflectivity dips
Green region: high reflectivity

From 2D Ag hole array





1st Brillouin zone

$$k_{sp} = \sqrt{\left(\frac{\omega}{c}\sin\theta\cos\varphi + \frac{2m\pi}{a}\right)^2 + \left(\frac{\omega}{c}\sin\varphi\cos\theta + \frac{2n\pi}{a}\right)^2}$$

J. Li et al, Appl. Phys. Lett. 92, 213106 (2008); K.C. Hui, Appl. Phys. Lett. 95, 063110 (2009)

Numerical vs analytical

- Many parameters are involved. Period, hole depth and radius, wavelength, incident angle, type of resonance modes, etc.

  Difficult to find the right condition.

- Experimental: databank.

- Numerical methods: time consuming, resource demanding, lack of overall picture, etc.

- Analytical: may be qualitative, broad picture and physical insight.

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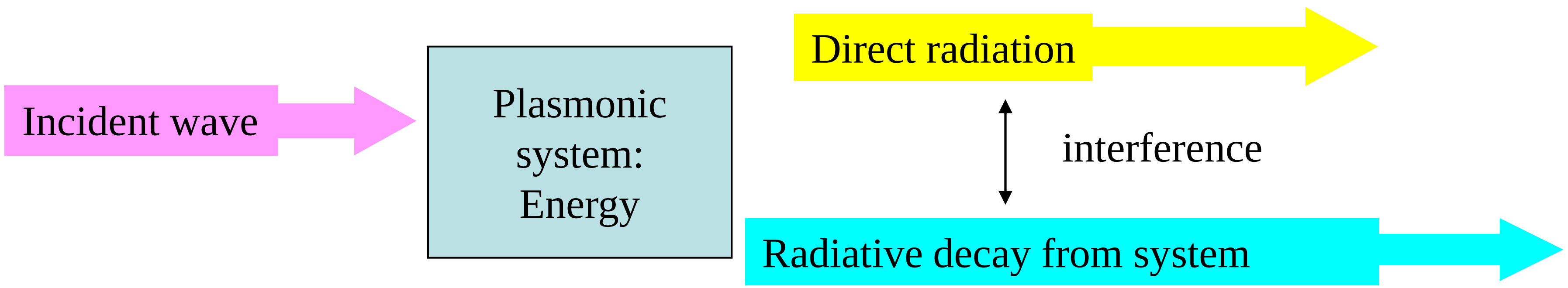
Coupled mode theory (CMT)



H.A. Haus, "Waves and Fields in Optoelectronics," 1984; S. Fan, "Photonic crystal theory: temporal coupled-mode formalism," Optical Fiber Telecommunications V A: Components and Subsystems, 2008; S. Fan et al, JOSA A 20, 569 (2003); L. Verslegers et al, JOSA B, 27, 1947 (2010); T.J. Seok et al, Nano Lett. 11, 2606 (2011); S.A. Maier, Opt. Exp. 14, 1957 (2006); J.B. Khurgin et al, APL 94, 1911106 (2009), APL 94, 101103 (2009), APL 95, 171103 (2009))

For single port, the mode amplitude, a, is given as:

$$\frac{da}{dt} = i\omega_o a - \frac{1}{2}\left(\Gamma_{abs} + \Gamma_{rad}\right)a + \kappa\sqrt{\frac{\Gamma_{rad}}{2}}s_+$$

Decay of SPP

Coupling from incident wave

Decay rates are very important!!!

ω_o = resonance frequency

$|a|^2$ = energy in the resonator

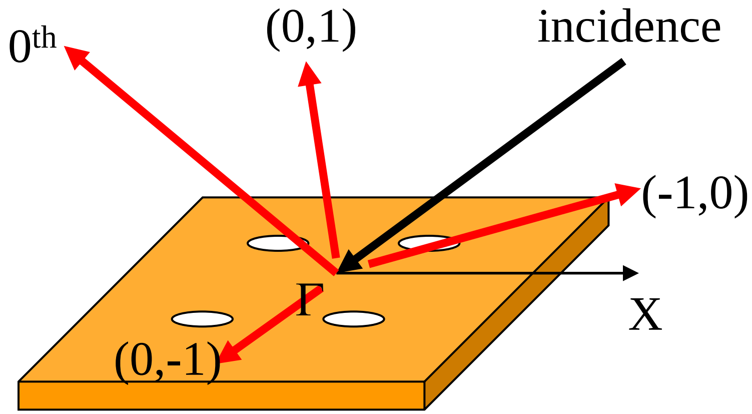
κ = coupling constant

$|s_+|^2$ = power carried by incident wave

Γ_{abs} = absorption rate

Γ_{rad} = radiative decay rate

$\Gamma_{tot} = \Gamma_{abs} + \Gamma_{rad}$



- Can be extended to multiple ports
- Can be applied to different dimensionalities and systems. For example, thin films, nanoparticles, etc.

- Solve for a

$$a = \frac{\kappa\sqrt{\frac{\Gamma_{rad}}{2}}\, s_+}{i(\omega-\omega_o)+\Gamma_{tot}/2}$$

- For (-1,0) SPP, single port CMT yields Fano-like reflectivity

outgoing

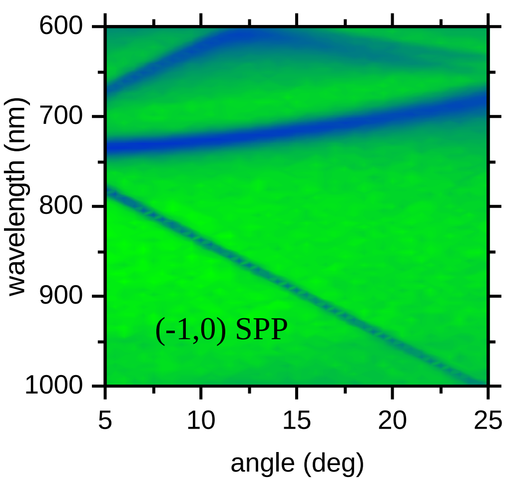
$$R = \frac{|s_-|^2}{|s_+|^2} = \left|\alpha + \kappa\sqrt{2}\,\frac{a}{s_+}\right|^2 = \left|\alpha + \frac{\kappa^2\frac{\Gamma_{rad}}{2}e^{i\varphi}}{i(\omega-\omega_o)+\Gamma_{tot}/2}\right|^2$$

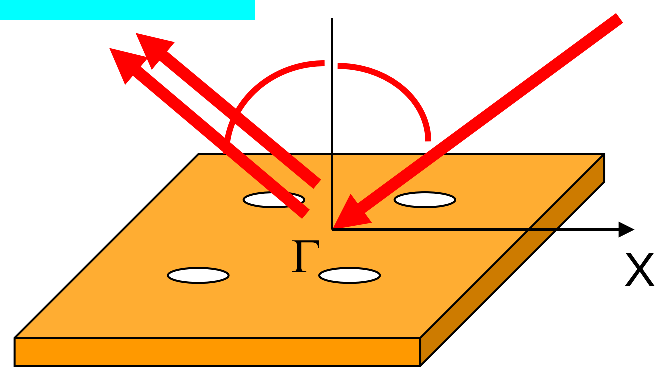
incoming

Direct reflection

Radiative decay of SPP

Specular reflection

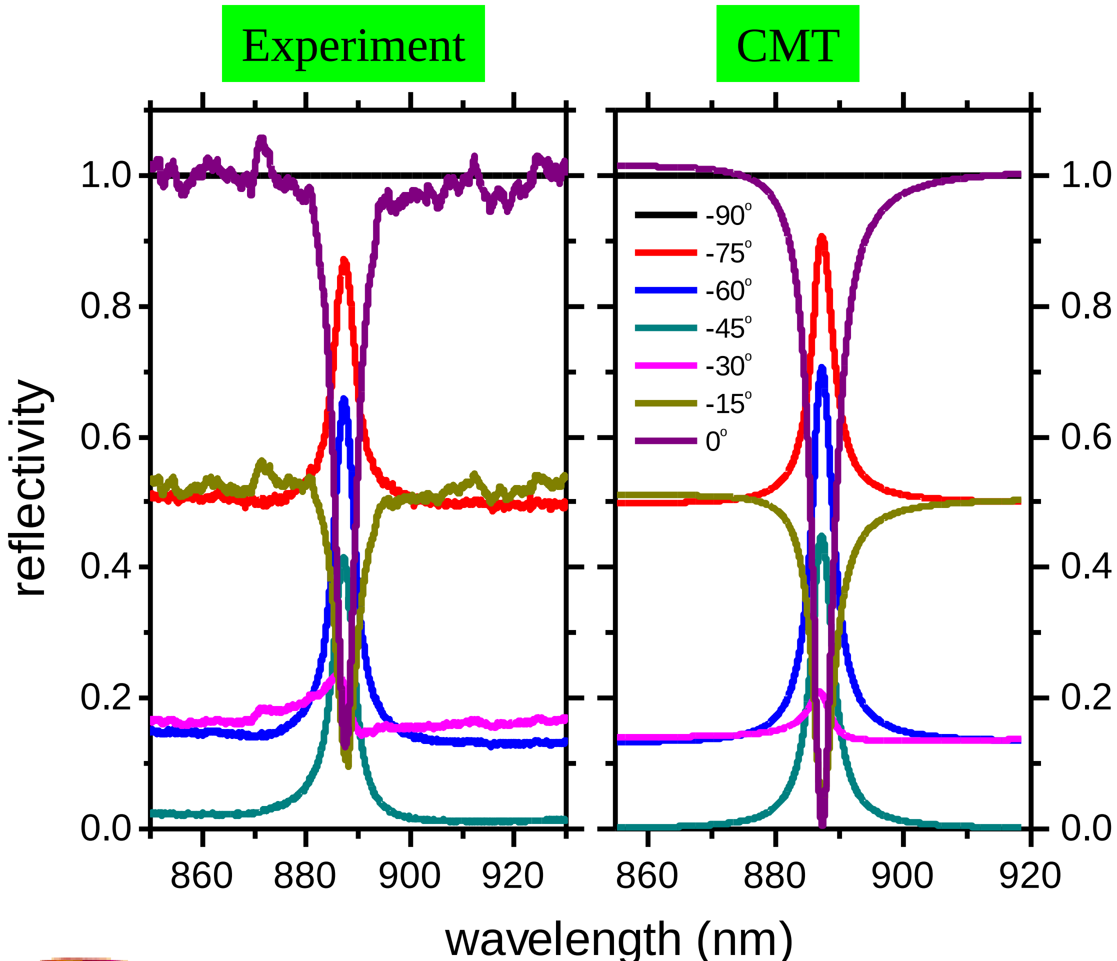


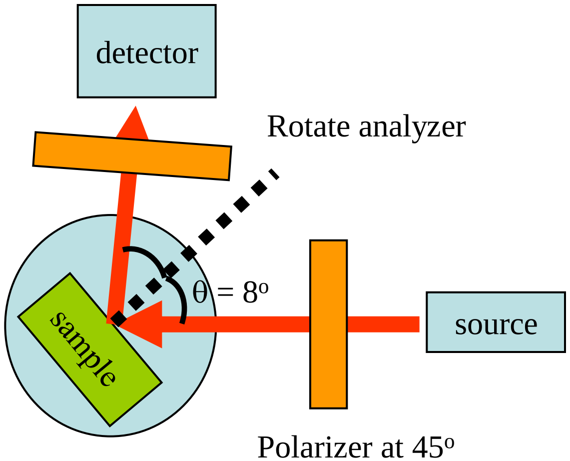


H.Y. Lo et al, OL 37,2736 (2012); H.Y. Lo et al APL (in press)

Polarization-dependent spectroscopy to verify CMT

Au 2D hole array, period = 760 nm, diameter = 210 nm





Best fit 0° spectrum and calculate, -15° → -90° spectra by CMT and Jones calculus

Normalized w.r.t. -90° spectrum

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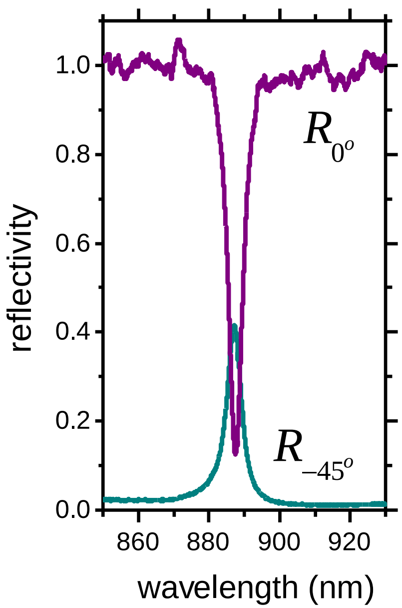
By conservation of energy

$$|s_+|^2 - |s_-|^2 = \Gamma_{abs}|a|^2 + \Omega_{Au}|s_+|^2$$

SPP absorption

Flat Au absorption



From fitting

0° reflectivity

From fitting

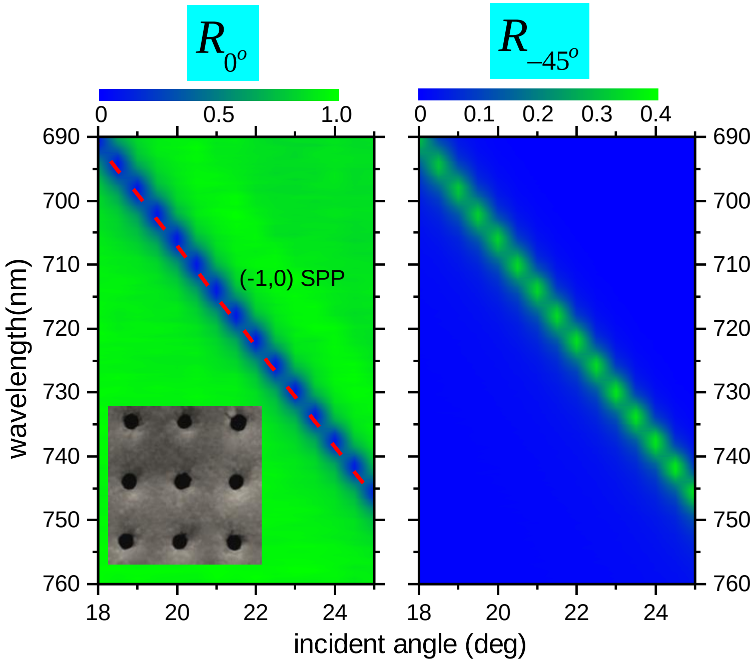
$$\Gamma_{abs} = \frac{\left(1-\Omega_{Au} - R_{0^o}\right)|\kappa^2\Gamma_{rad}|}{8R_{-45^o}}$$

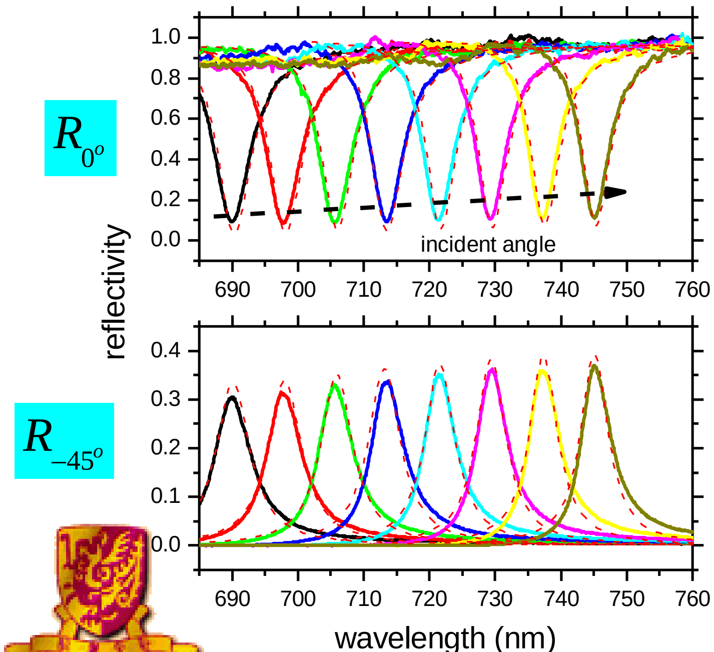
-45° reflectivity

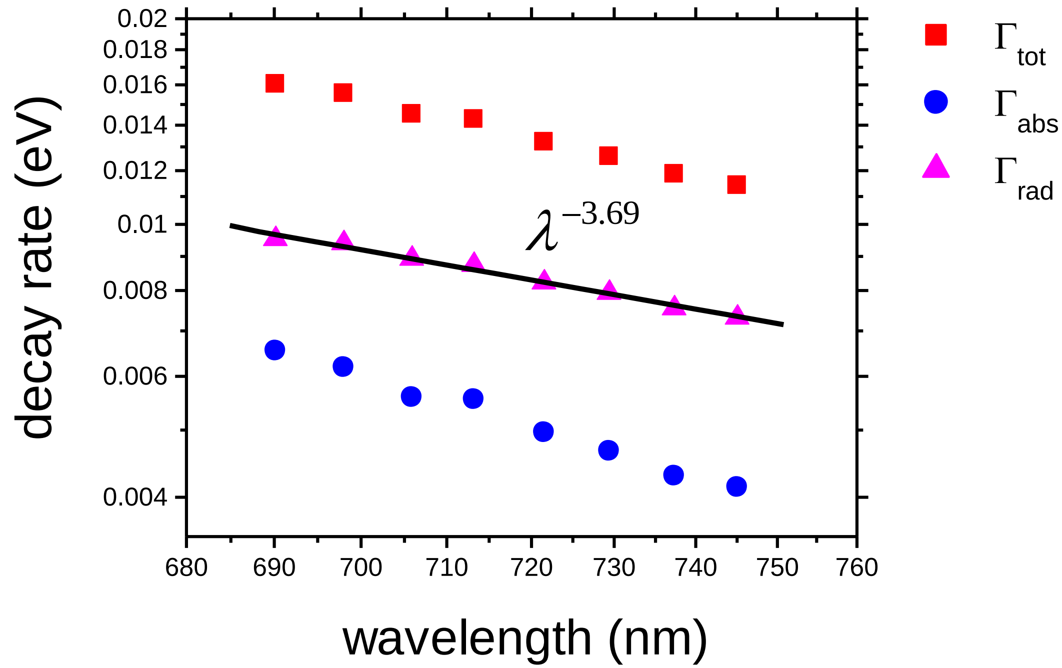
$$\Gamma_{rad} = \Gamma_{tot} - \Gamma_{abs}$$

From fitting

P = 515 nm, hole depth H and diameter D = 280 and 140 nm

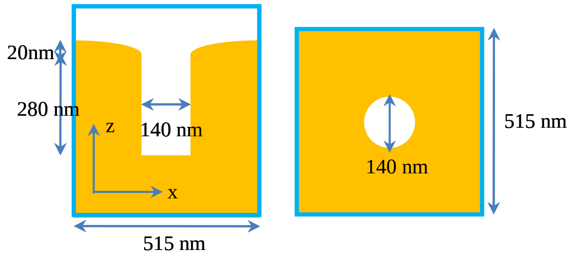


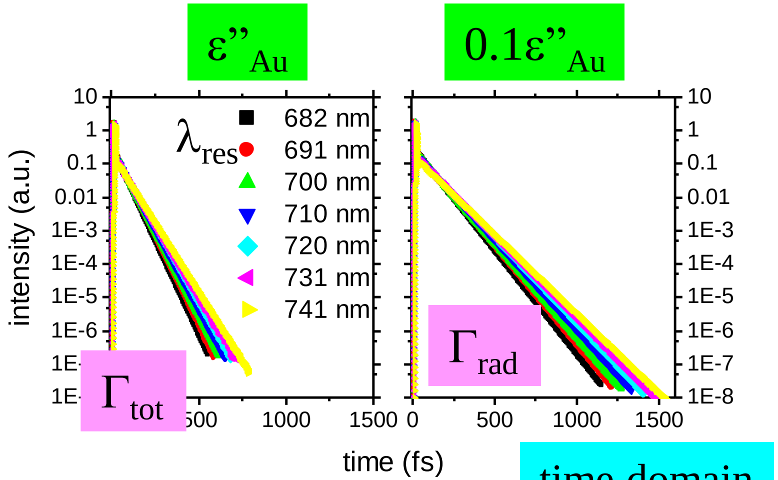


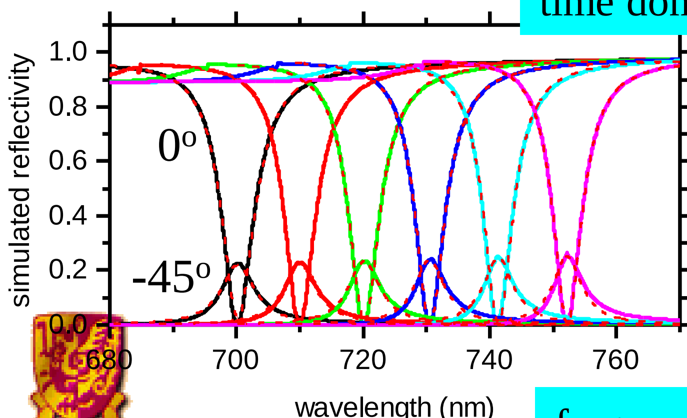


FDTD simulation

Unit cell

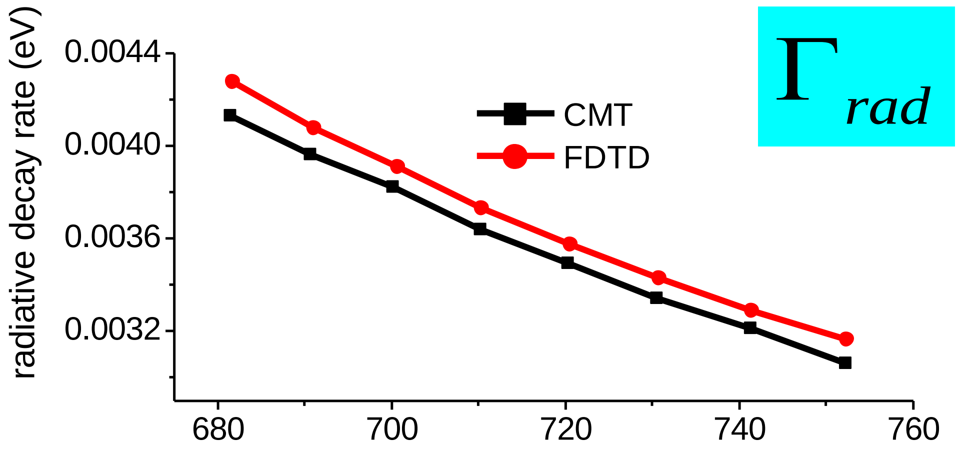


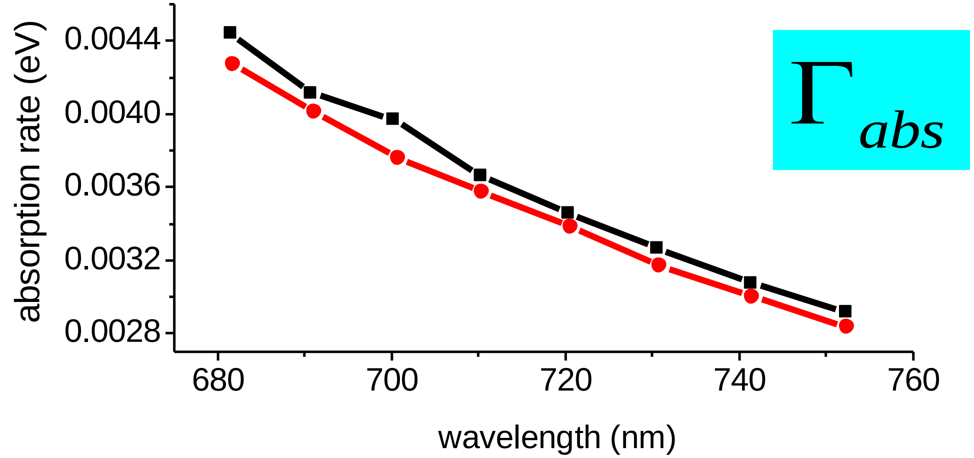




(-1,0) SPP

frequency domain





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Maximize the field strength?

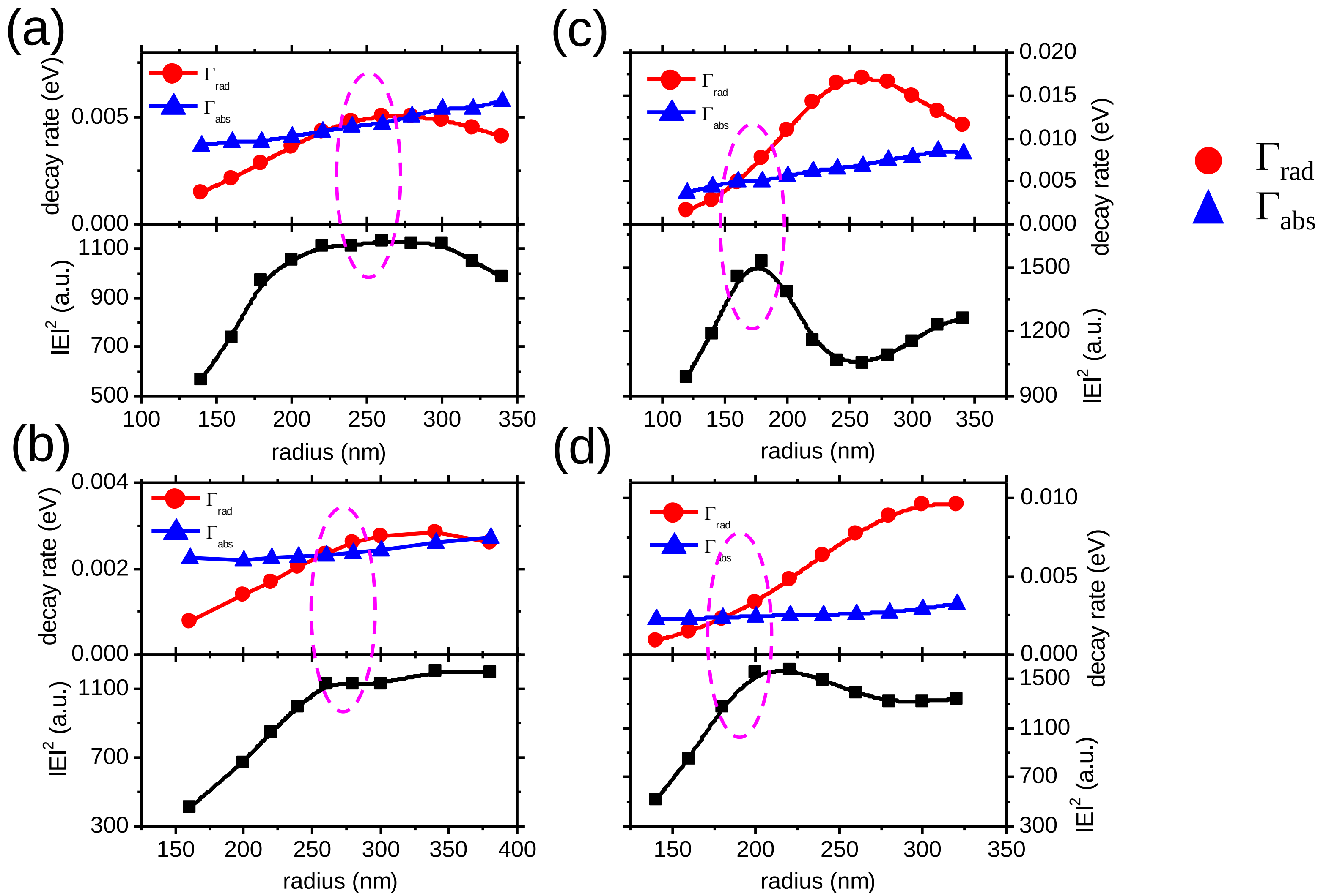
Energy of SPP, $|a|^2$, is given as:

$$|a|^2 = \frac{\kappa^2 \frac{\Gamma_{rad}}{2} |s_+|^2}{(\omega - \omega_o)^2 + \left(\frac{\Gamma_{tot}}{2}\right)^2} \quad \Rightarrow \quad |E|^2 \propto \frac{|a|^2}{V_{eff}}$$

One special case, if Γ_{abs} and κ do not vary much, $|a|^2$ is maximal when $\Gamma_{abs} = \Gamma_{rad}$.

$$\frac{d|a|^2}{d\Gamma_{rad}} = 0$$

2D Au periodic arrays (-1,0) SPP: FDTD



(a) P = 760 nm and depth = 60 nm, (b) P = 900 nm and depth = 60 nm, (c) P = 760 nm and depth = 120 nm, and (d) P = 900 nm and depth = 120 nm.

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P- and s-reflectivity

- P- and s-reflections and p-s phase difference

p: $$r_p = r_p' e^{i\phi_p} = \alpha + \frac{\kappa^2 \frac{\Gamma_{rad}}{2} e^{i\varphi}}{i(\omega-\omega_o) + \frac{\Gamma_{tot}}{2}}$$

by CMT

s: $$r_s = r_s' e^{i\phi_s} = \beta$$

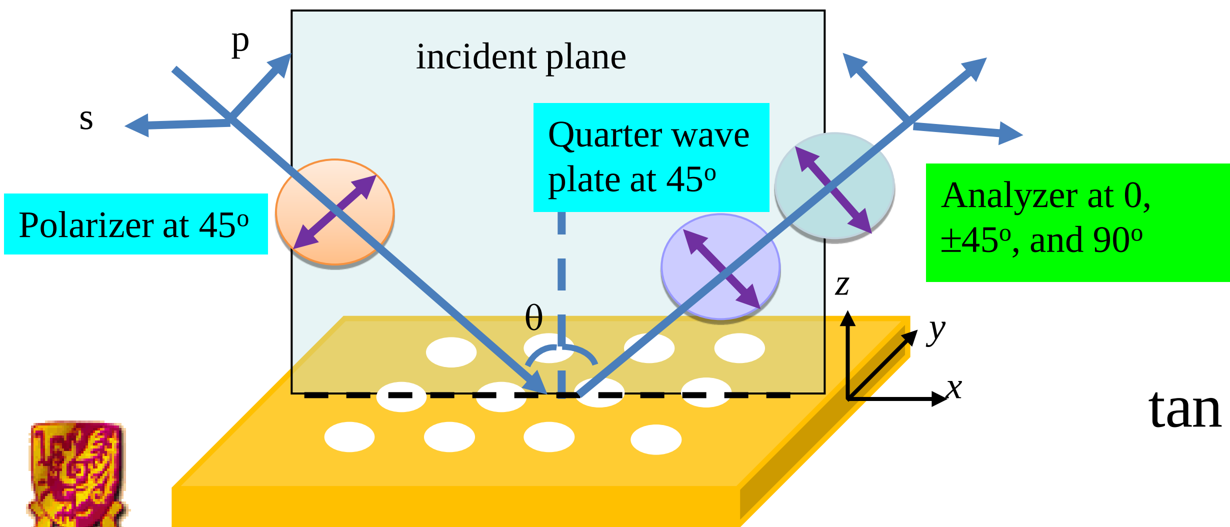
direct reflection

radiative decay

$$\phi = \phi_p - \phi_s$$

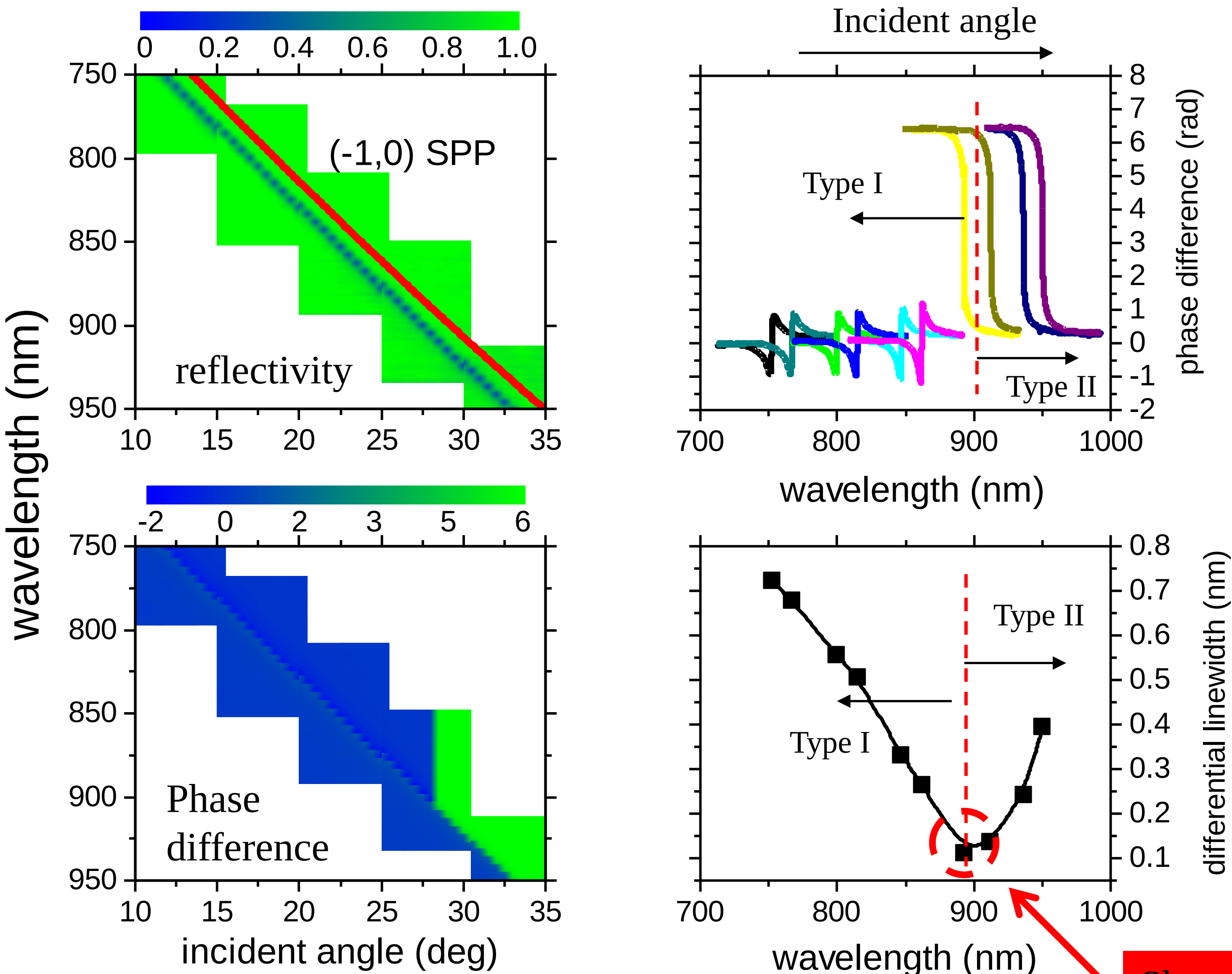
P-s phase difference

- Phase difference measurement by angle-resolved phase quadrature common-path interferometry



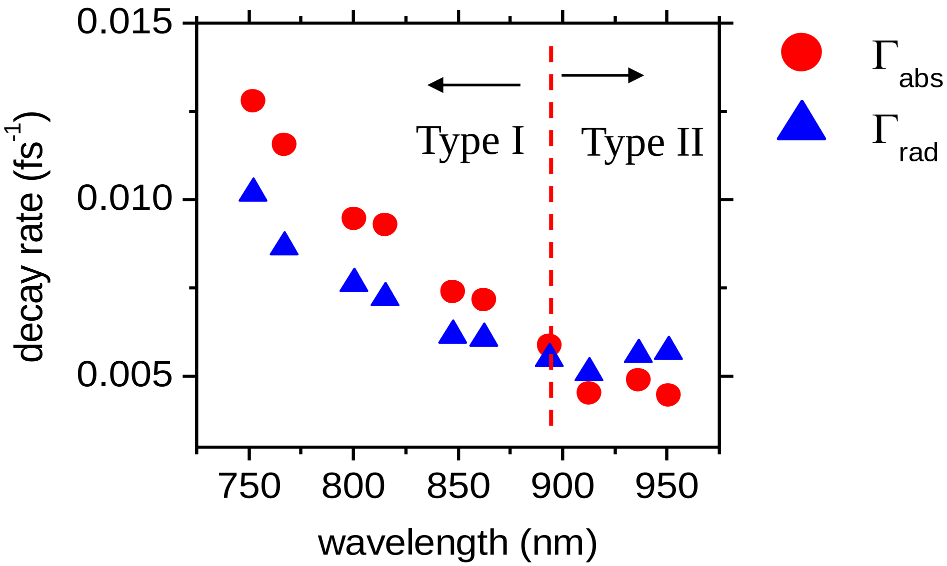
By Jones matrix

$$\tan\phi(\lambda) = \frac{I_0(\lambda) - I_{90}(\lambda)}{I_{+45}(\lambda) - I_{-45}(\lambda)}$$



Sharpest phase jump
Good for SPR sensing

Period = 600 nm, hole depth and radius = 80 and 60 nm



Transition at $\Gamma_{rad} \approx \Gamma_{abs}$

By Jones matrices to calculate the phase difference

$$\begin{bmatrix} r_p \\ r_s \end{bmatrix} = \begin{bmatrix} \alpha + \dfrac{\kappa^2 \dfrac{\Gamma_{rad}}{2} e^{i\varphi}}{i(\omega - \omega_o) + \dfrac{\Gamma_{tot}}{2}} \\ \beta \end{bmatrix}$$

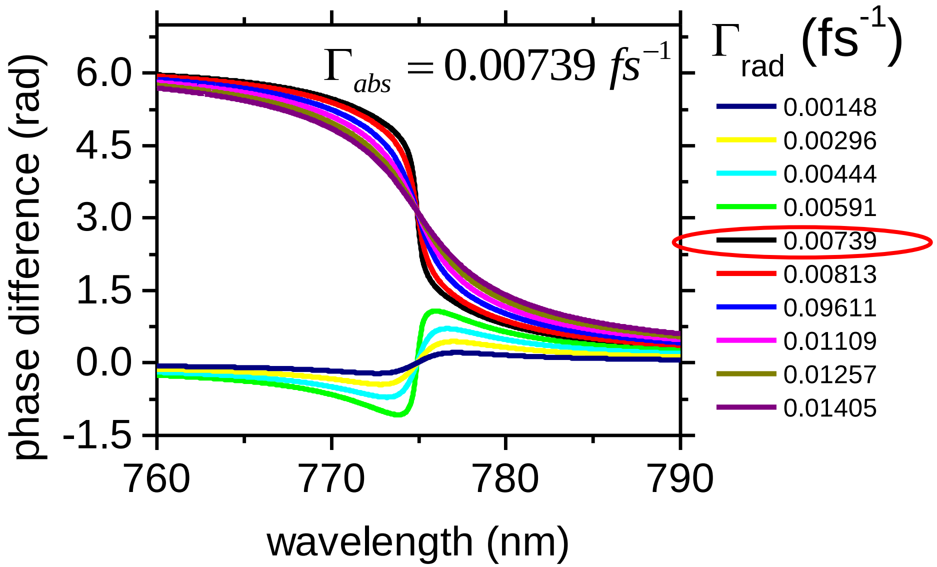
$\alpha = \beta = 0.95$

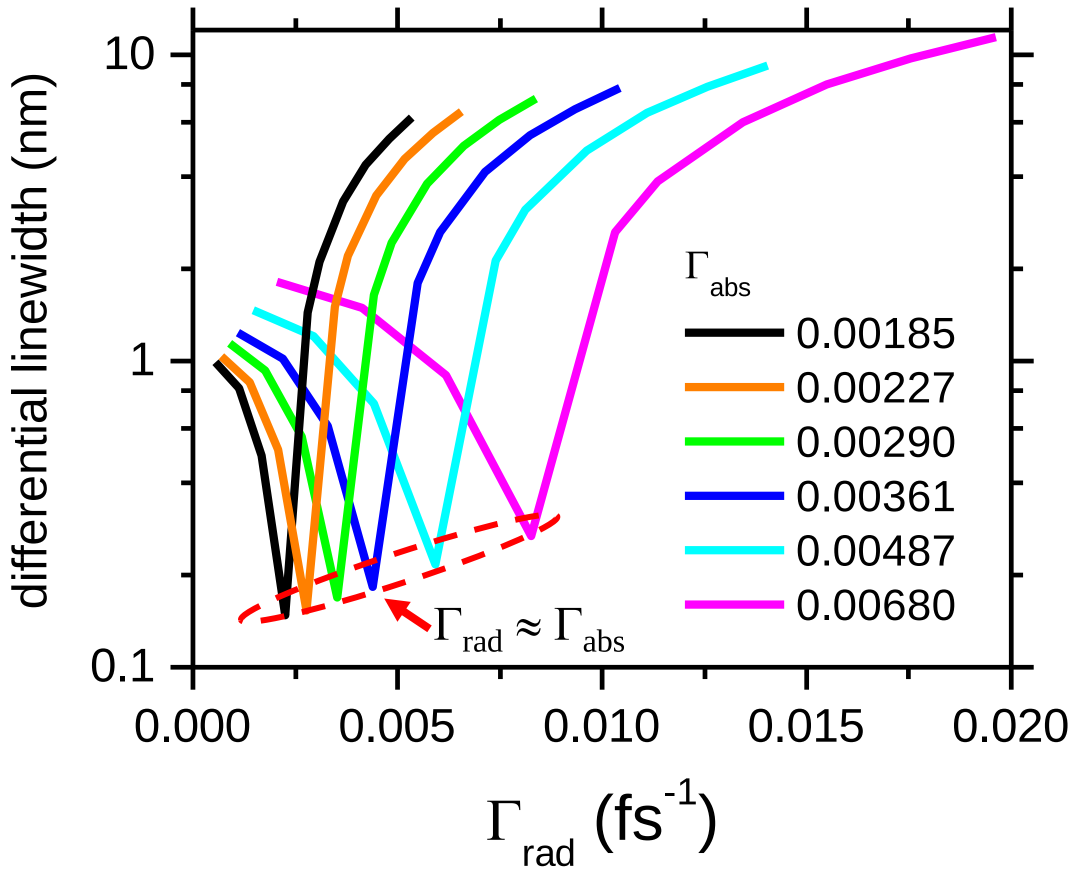
$\kappa = \sqrt{2}$

$\varphi = \pi$

$\Gamma_{abs} = 0.00739\ fs^{-1}$

Transition at $\Gamma_{rad} \approx \Gamma_{abs}$





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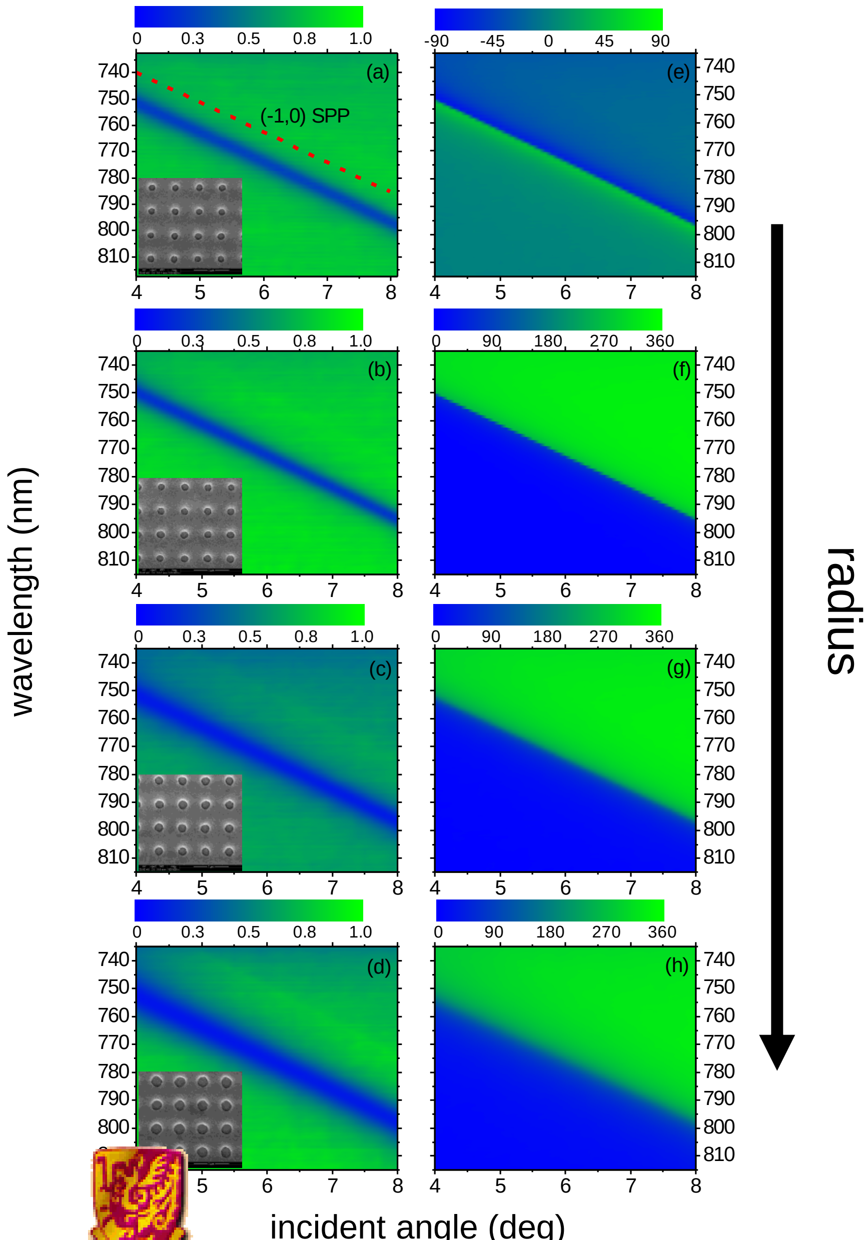
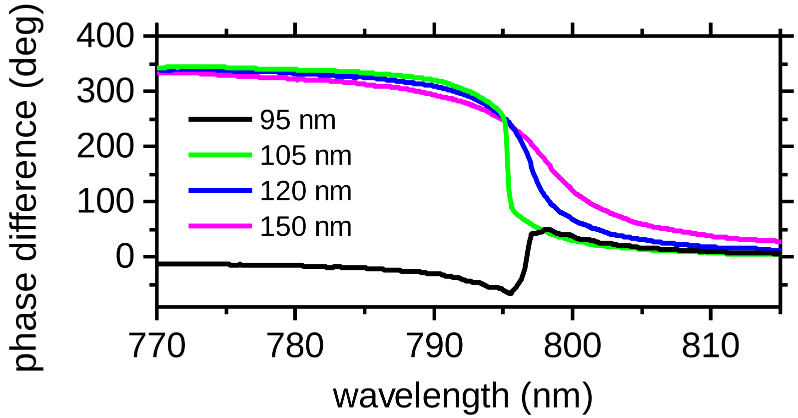
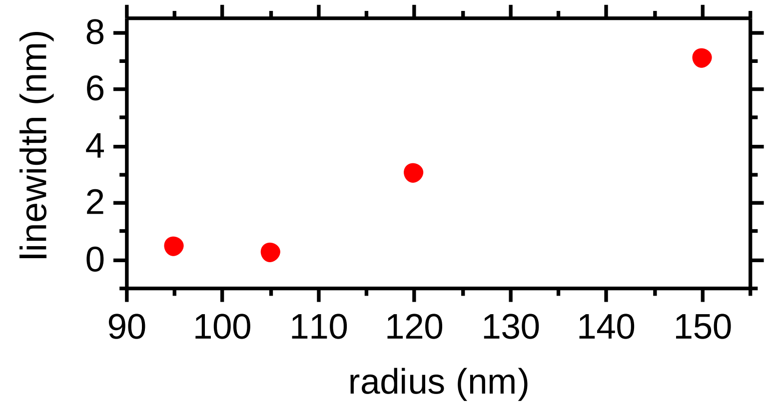


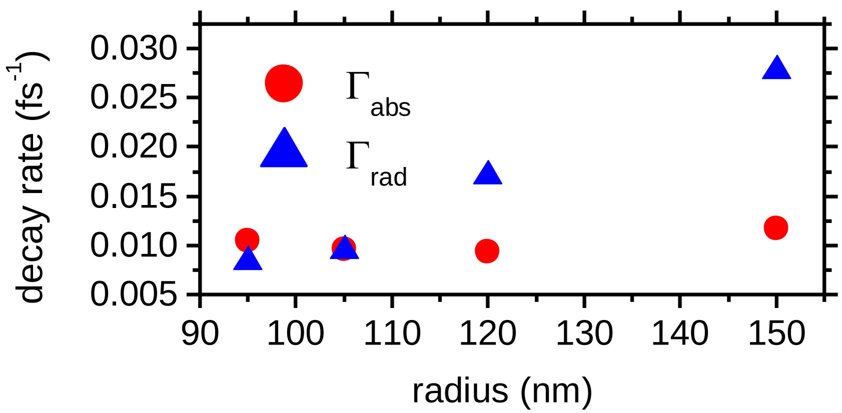
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(-1,0) SPP
wavelength (nm)
incident angle (deg)
radius



phase difference (deg)
wavelength (nm)
95 nm
105 nm
120 nm
150 nm



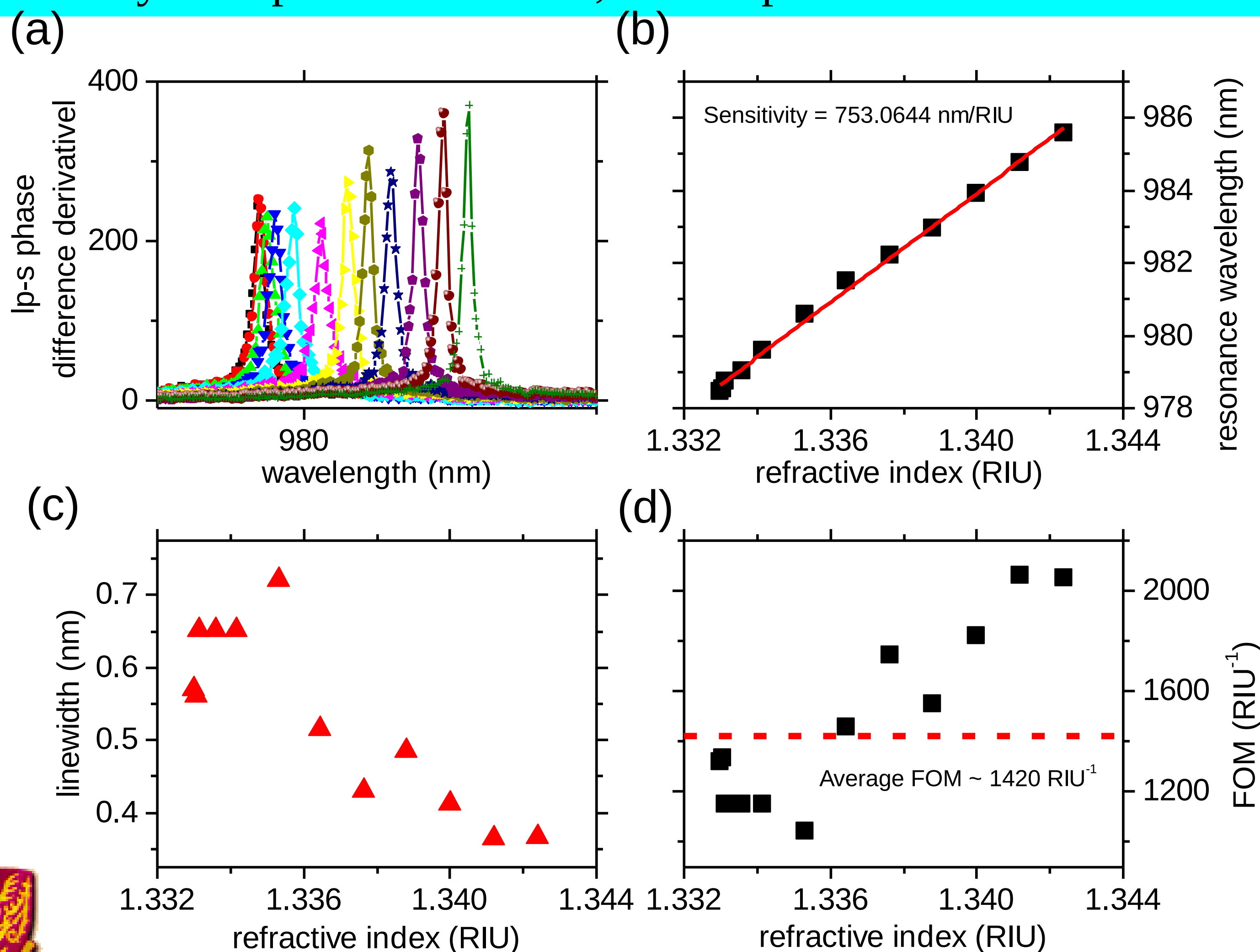
linewidth (nm)
radius (nm)



decay rate (fs^{-1})
radius (nm)
Γ_{abs}
Γ_{rad}

Phase-based SPR

2-D Au array with period = 750 nm, hole depth = 100 nm and radius = 80 nm

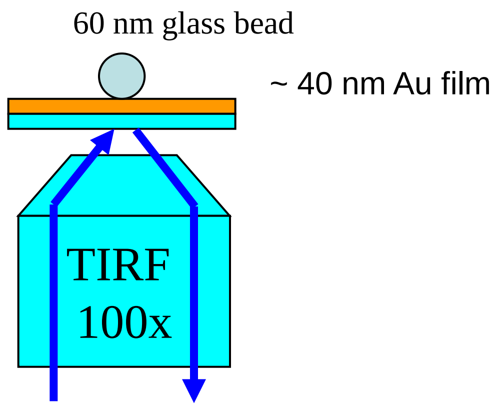


15 times higher than commercial SPR

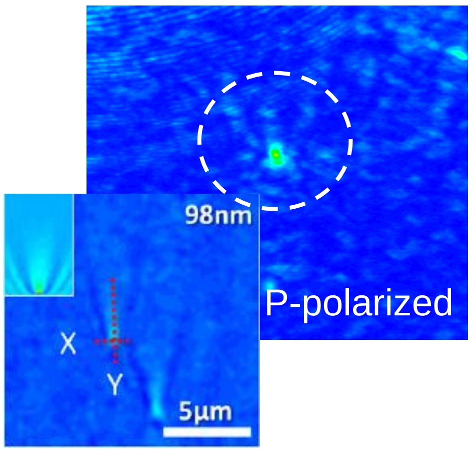
$$FOM = \frac{sensitivity}{linewidth}$$

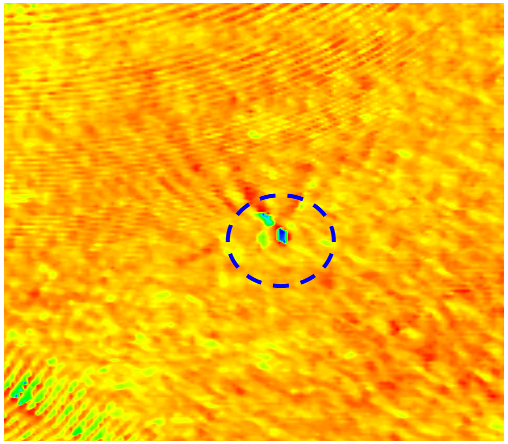
Something extra: SPR imaging

- CMT can be applied to attenuated total reflection thin film as well
- Change Au film thickness to tune $\Gamma_{rad} \approx \Gamma_{abs}$ (Raether, Springer)

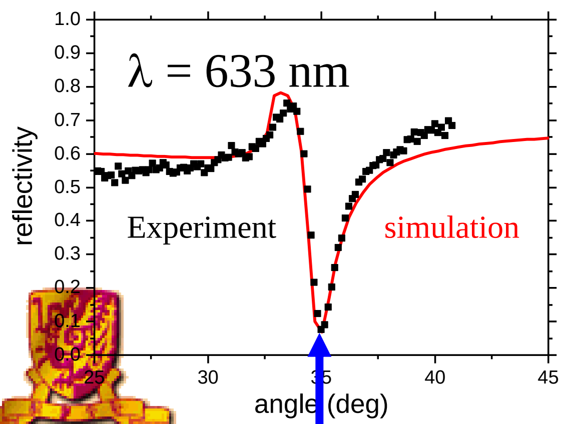


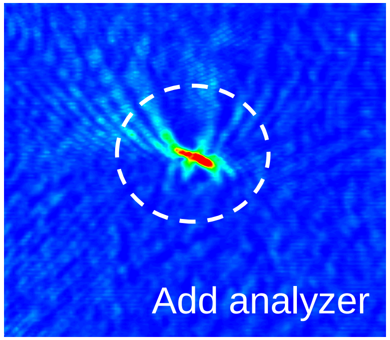
angle-resolved phase quadrature
common-path interferometry

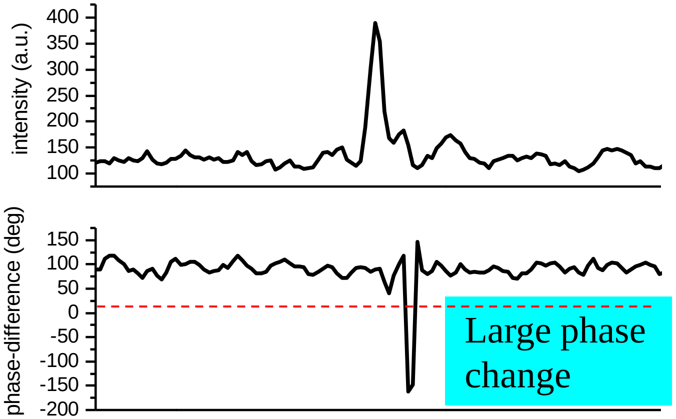




Wang et al, PNAS, 107, 16028 (2010)







Conclusions

- Coupled mode theory (CMT) is a useful analytical tool for studying the optical properties of period metallic arrays.
- Determine the absorption and radiative decay rates of SPPs.
- Under some special condition, the field strength is strongest when absorption rate = radiative decay rate.
- Sharpest p-s phase difference phase jump when absorption rate = radiative decay rate. Possible to have very high FOM.

Radiative decay of SPPs

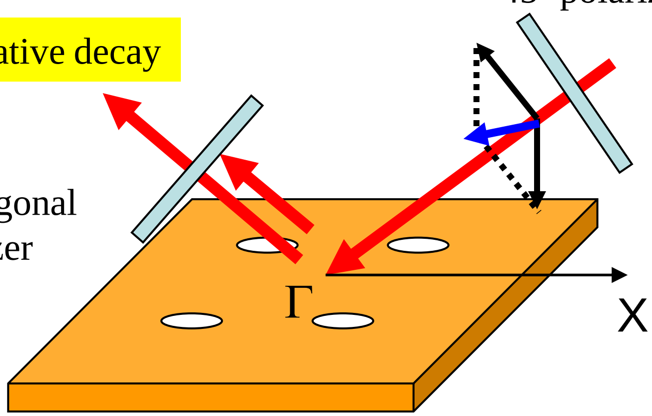
For (-1,0) SPP, the Fano-like reflectivity contains radiative decay

$$R = \frac{|s_-|^2}{|s_+|^2} = \left|\alpha + \kappa\sqrt{2}\frac{a}{s_+}\right|^2 = \left|\alpha + \frac{\kappa^2 \frac{\Gamma_{rad}}{2} e^{i\varphi}}{i(\omega-\omega_o) + \Gamma_{tot}/2}\right|^2$$

outgoing ($|s_-|^2$), incoming ($|s_+|^2$)

Direct reflection (α)

Radiative decay of (-1,0) SPP



By Jones matrix

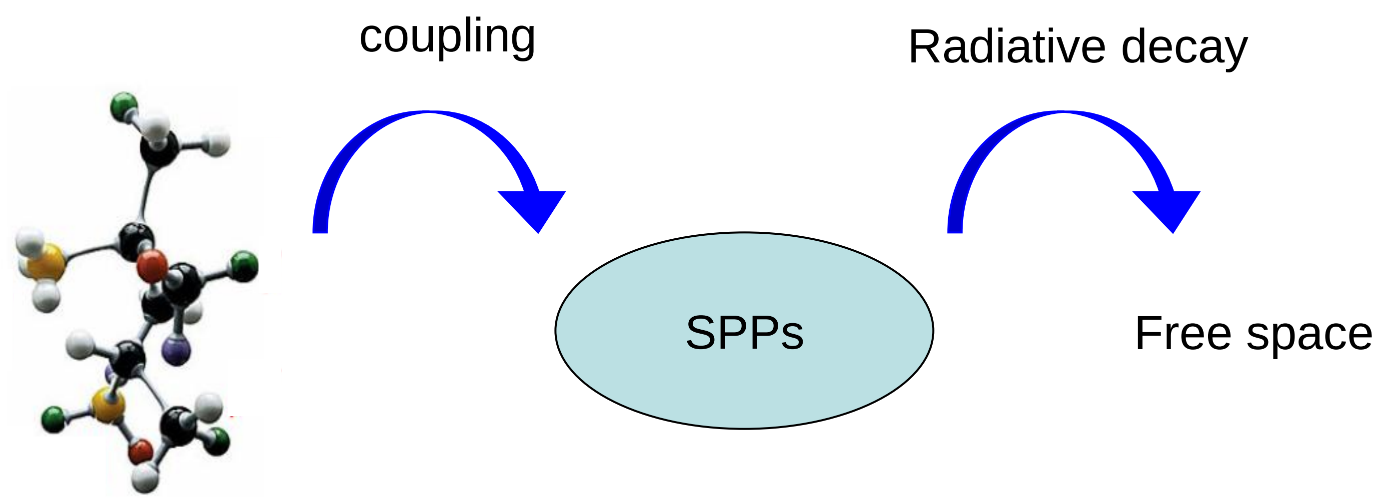
Lorentzian

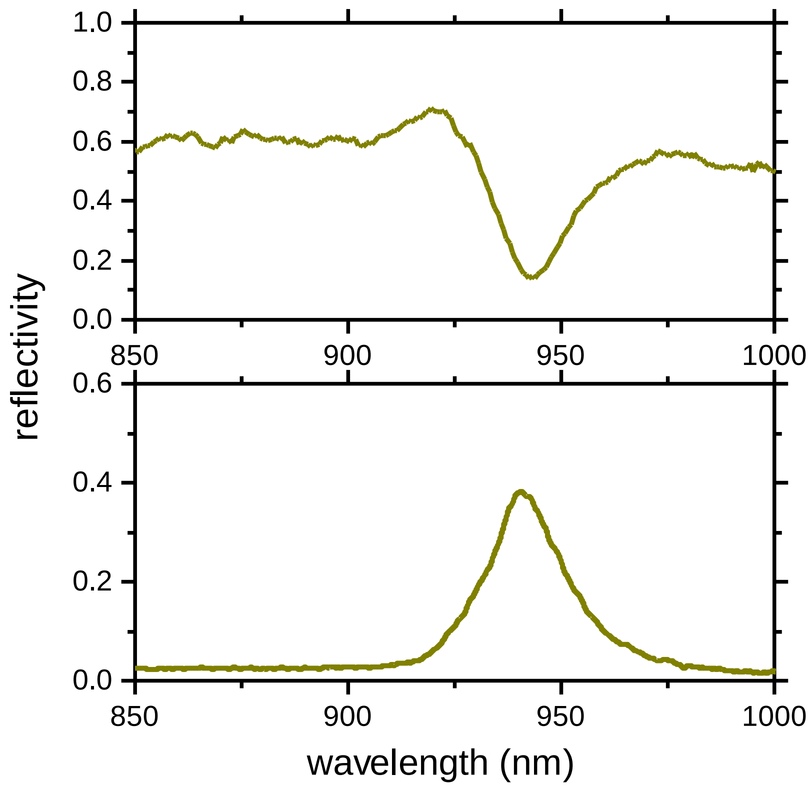
$$\frac{1}{4}\frac{\left(\kappa^2\Gamma_{rad}/2\right)^2}{\left(\omega-\omega_o\right)^2 + \left(\Gamma_{tot}/2\right)^2}$$

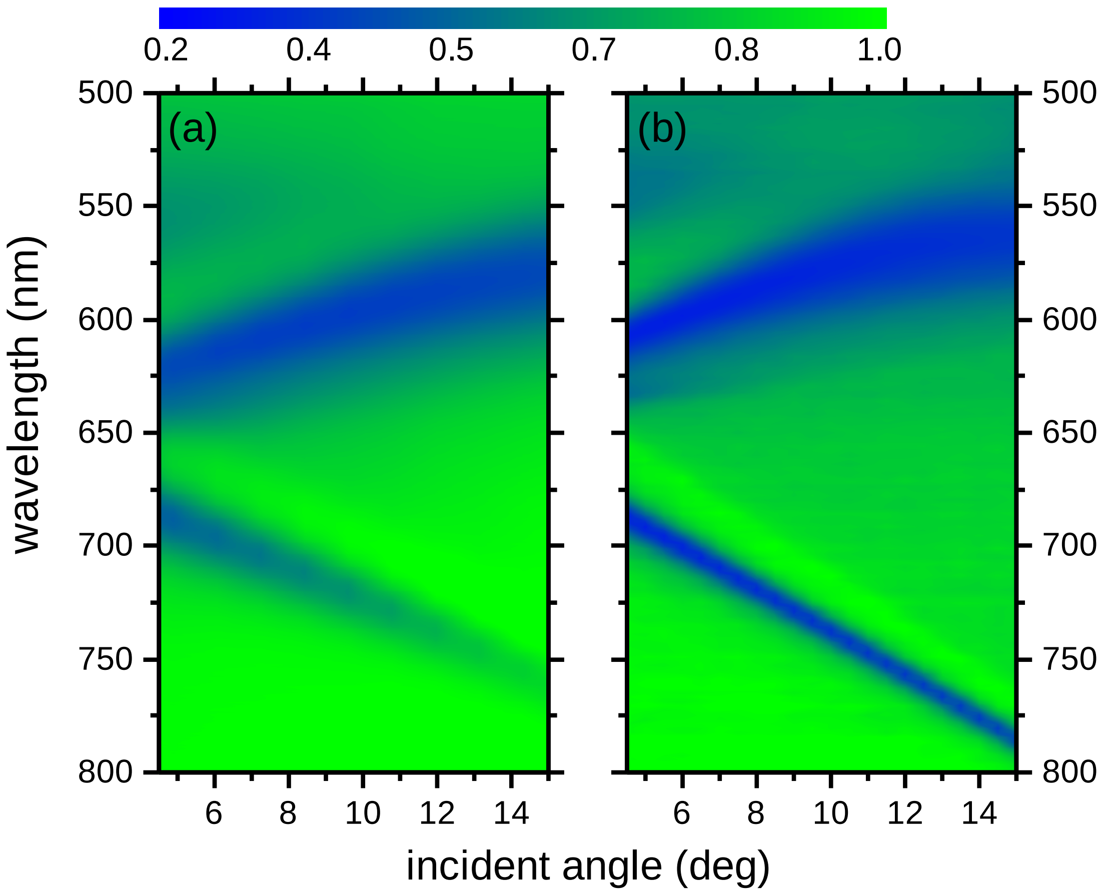
Power ratio between radiative scattering and incident field

Importance of radiative scattering

For Raman and fluorescence

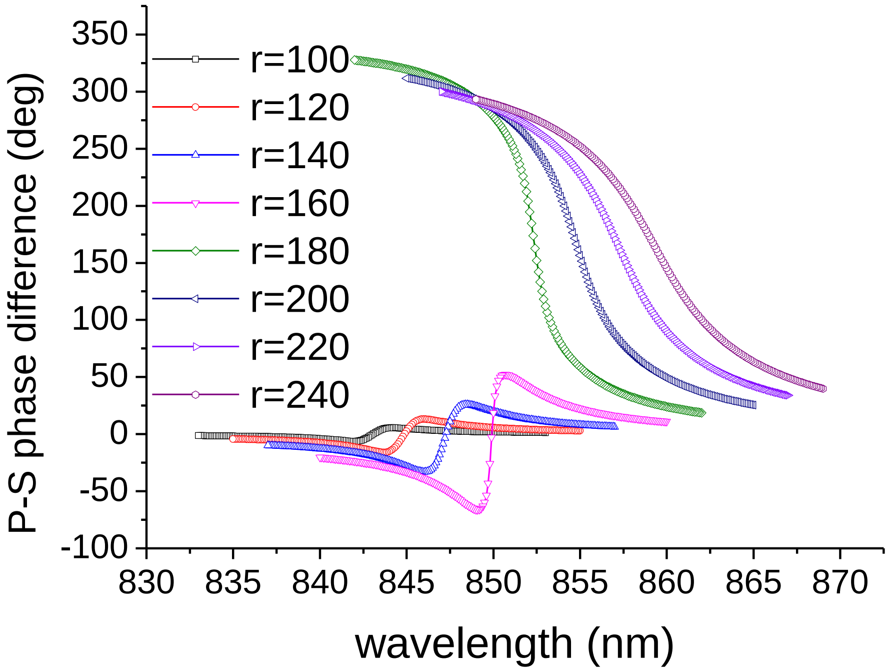



1.0
0.8
0.6
0.4
0.2
0.0
850
900
950
1000
0.6
0.4
0.2
0.0
850
900
950
1000
reflectivity
wavelength (nm)

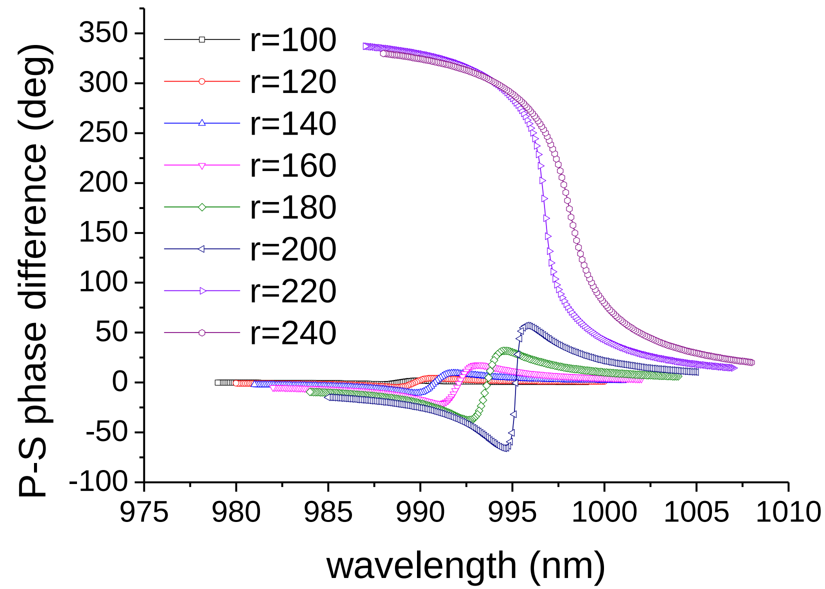

0.2
0.4
0.5
0.7
0.8
1.0
(a)
(b)
wavelength (nm)
500
550
600
650
700
750
800
6
8
10
12
14
incident angle (deg)

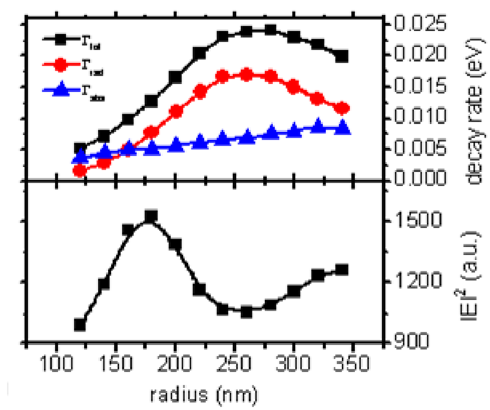
Phase change

Period =760nm, λ ~ 860nm

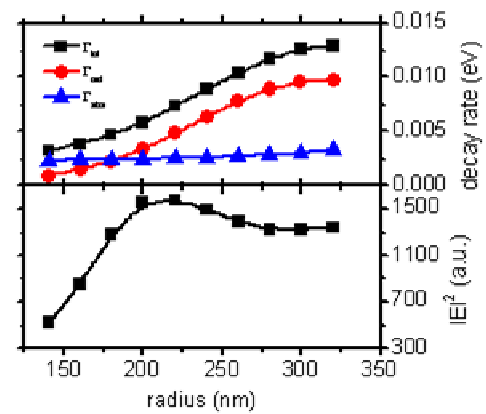


Period =900nm, λ ~ 1000nm





Hole depth = 120 nm



Can we relate Γ_{abs} and Γ_{rad} wavelength and geometry?

We have for 2D hole arrays

- Combined quasi-static model, FDTD, and experiment to show

$$\Gamma_{tot} = \Gamma_{abs} + \Gamma_{rad}$$

$$\Gamma_{abs} = \frac{1.3 \times 10^7}{\lambda^{3.3}}$$

$$\Gamma_{rad} \approx \frac{6\pi^6 \lambda_p c}{P} \left\{ \frac{R^3 H^2}{\lambda^6} + 5\pi^2 \frac{R^{4.3} H^{2.7}}{\lambda^8} \right\}$$

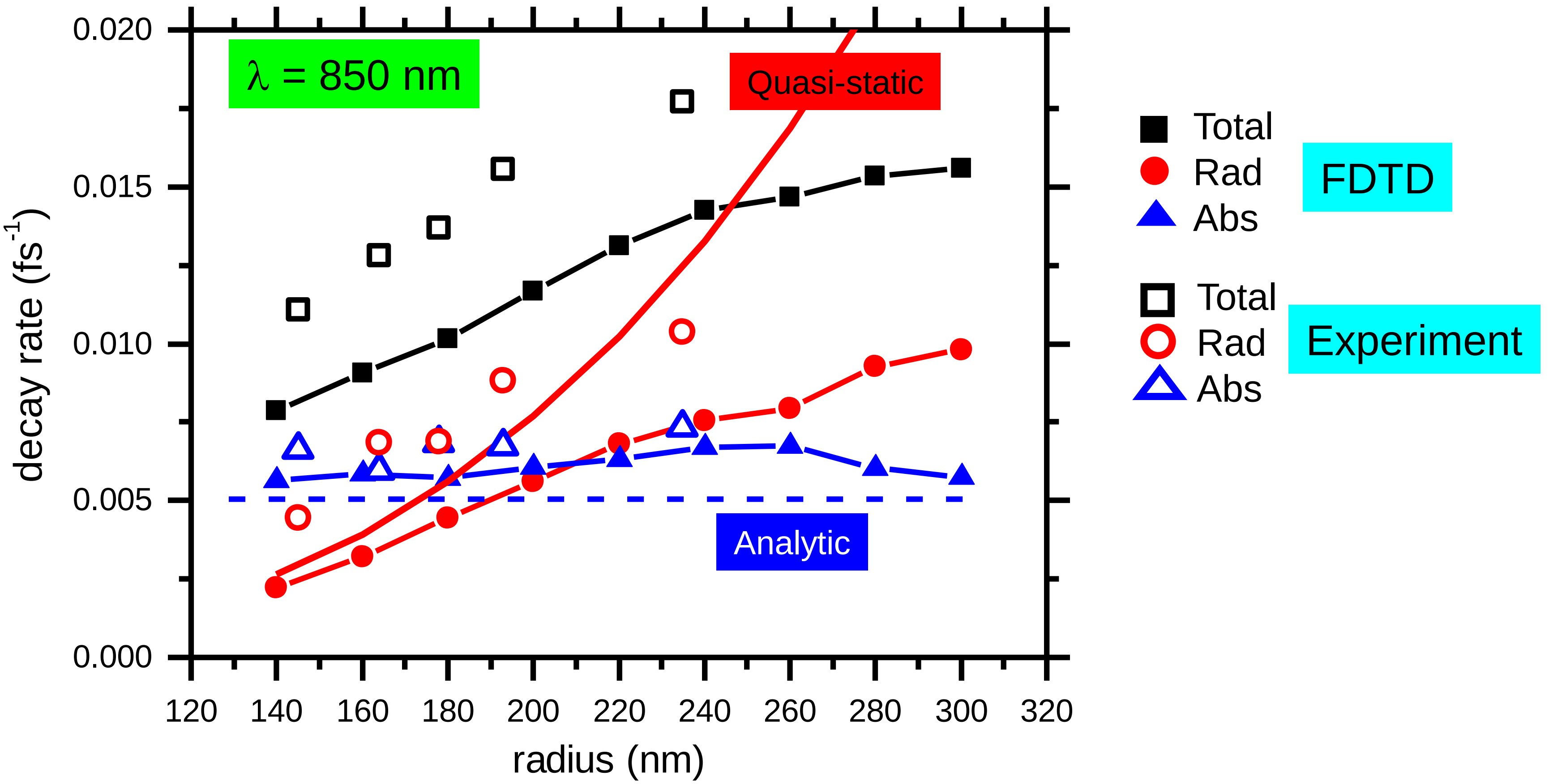
Both are functions of period (P), hole radius (R), and hole depth (H)

Give us guideline to control the decay process by using geometry

J. Li et al, Appl. Phys. Lett. 94, 183112 (2009); D.Y. Lei et al, ACS Nano 4, 432 (2010).

One result

2D Au periodic arrays, period = 760 nm and hole depth = 60 nm



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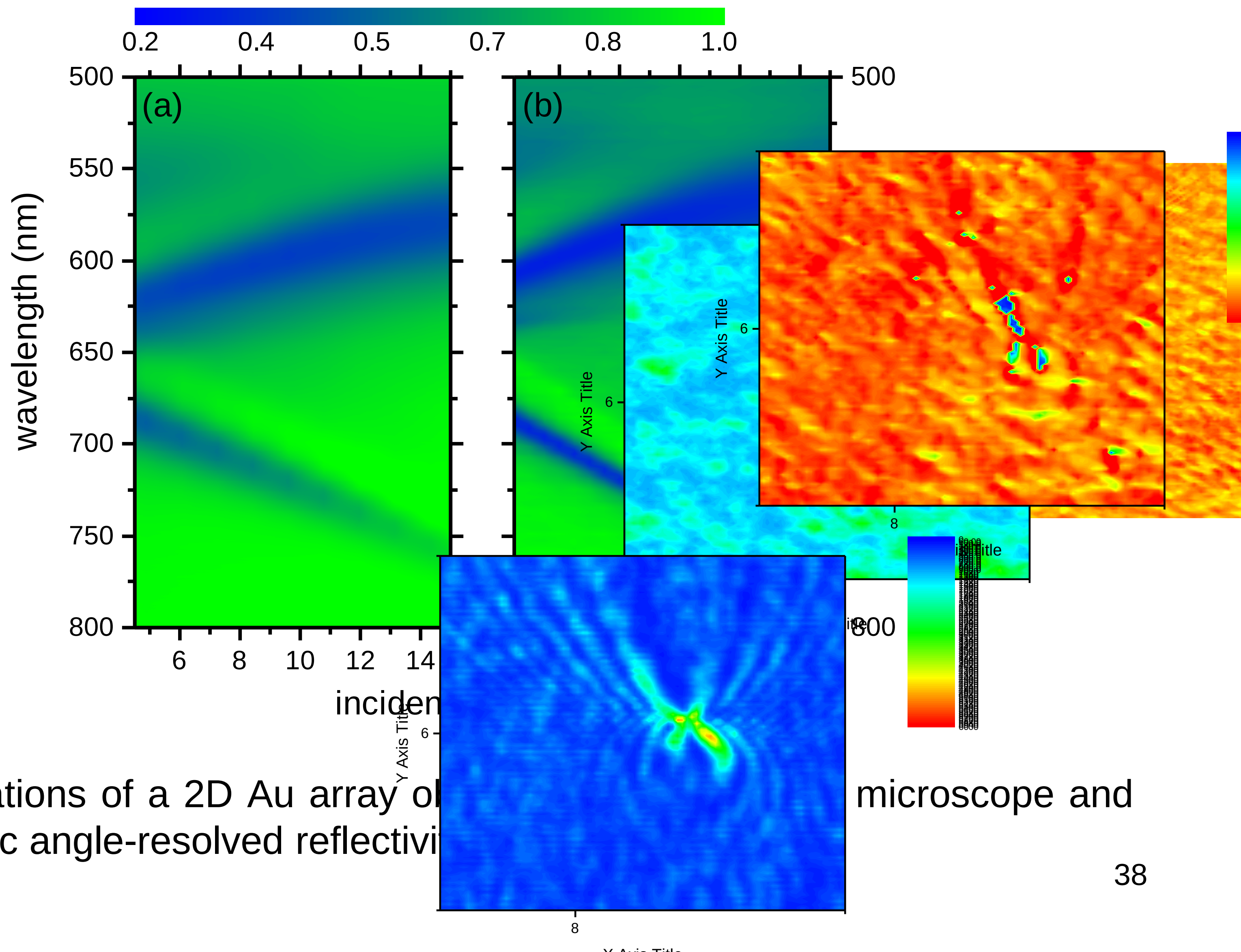
ZL. Cao et al, Opt. Lett. (submitted)



Dispersion relations of a 2D Au array o… microscope and (b) macroscopic angle-resolved reflectivi…