IAS Program on Frontiers of Soft Matter Physics: Tutorial

From Brownian to Driven and Active Dynamics of Colloids: Energetics and Fluctuations Part I

Masaki Sano

Department of Physics

The University of Tokyo

Outline of Talk

Part I:

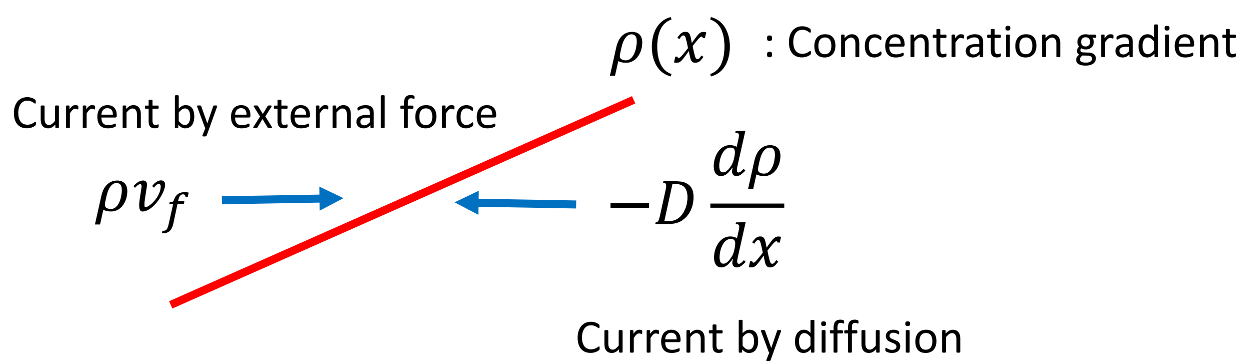
- Introduction to Brownian motion, Fluctuation Dissipation Theorem
- Stochastic Energetics (by K. Sekimoto, 1997)
- Fluctuation Theorems, Jarzynski Equality, etc. (1993-1996) → (Tuesday: by Hyunggyu Park)
- Measuring energy dissipation in small systems

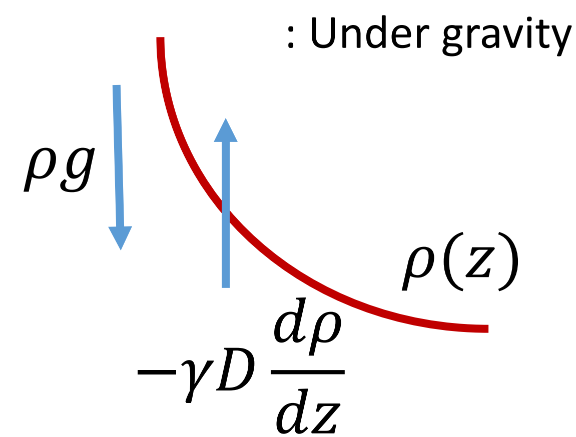
Part II:

- Information Feedback control and thermodynamics
- Self-propelled particles, Active Matter

Einstein's Derivation of the Relation for Brownian Motion

- Consider colloidal suspension in an equilibrium state under the balance between external field and diffusion.





Total current: $$j = \rho v_f + j_D = \rho v_f - D\frac{\partial \rho}{\partial x}$$

Equilibrium condition: $$j = 0$$

$$\Rightarrow \quad \rho v_f = D\frac{\partial \rho}{\partial x} \qquad (1)$$

- Osmotic pressure due to particle concentration

$$p = k_B T \rho$$

- Force due to osmotic pressure balances with the external force f

$$\frac{\partial p}{\partial x} = \rho f \quad \Rightarrow \quad k_B T \frac{\partial \rho}{\partial x} = \rho f \qquad (2)$$

Stokes's law of drag force: $f = \gamma v_f$

$\gamma = 6\pi\eta a$: Drag coefficient

From eq. (1) and (2)

$$\frac{k_B T}{\gamma}\frac{\partial \rho}{\partial x} = \rho v_f = D\frac{\partial \rho}{\partial x}$$

Einstein-Stokes Relation:

$$\boxed{D = \frac{RT}{N_A}\frac{1}{6\pi\eta a} = \frac{k_B T}{\gamma}}$$

Mesoscopic relation connecting between atomic scale and macroscopic scale

Brownian motion and Langevin equation

- Langevin equation

$$\dot{v}(t) = -\gamma v(t) + \frac{\xi(t)}{m} \quad (1)$$

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle = 2\epsilon\delta(t-t') \quad (2)$$

Integrating this equation

$$v(t) = v(t_0)e^{-\gamma(t-t_0)} + \frac{1}{m}\int_{t_0}^{t} e^{-\gamma(t-s)}\xi(s)ds$$

Multiplying v(t) and integrating it,

$$\langle v(t)^2 \rangle = \frac{\epsilon}{m^2\gamma}(1-e^{-2\gamma t}) + \langle v^2 \rangle e^{-\gamma t}$$

$t \to \infty \qquad \langle v(t)^2 \rangle = \epsilon/(m^2\gamma) \quad \Rightarrow \quad m\langle v(t)^2 \rangle = k_B T$

Using equipartition

$$\therefore \quad \boxed{\epsilon = k_B T m \gamma}$$

Fluctuation ∝ Dissipation

- Diffusion coefficient:

$$D \equiv \lim_{t\to\infty} \frac{x(t)^2}{2t} = \int_0^t \int_0^t \langle v(t_1)v(t_2)\rangle dt_1 dt_2$$

$$\Rightarrow \quad D = \frac{\epsilon}{(m\gamma)^2} = \frac{k_B T}{m\gamma} = \mu k_B T$$

$\mu \equiv 1/(m\gamma)$: mobility

FDT (Fluctuation Dissipation relation of 1st kind)

$$\mu = \frac{1}{k_B T}\int_0^\infty \langle v(0)v(t)\rangle dt$$

Integrating (2) gives

FDT (Fluctuation Dissipation relation of 2nd kind)

$$m\gamma = \frac{1}{2k_B T}\int_0^\infty \langle \xi(0)\xi(t)\rangle dt$$

Theories on Non-equilibrium Systems

Classical mechanics

Newton

Quantum mechanics

Shrödinger

Particle level

Non-equilibrium statistical mechanics

Liouville equation

BBGKY-hierarchy

Particle level

Time dependent correlation functions

Kinetic theory

Boltzmann equation

Master equation

Langevin equation

Fokker-Planck equation

Mesoscopic level

Kubo relation

Non-equilibrium thermodynamics

Linear flux force relations

Onsager relations

Navier-Stokes equation

Macroscopic level

Stochastic Energetics

by K. Sekimoto, JPSJ (1997)

See also, Ken Sekimoto, "Stochastic Energetics", Lecture Notes in Physics 799. (Springer)

Underdamped Langevin equation:

External control parameter

$$\frac{d^2x}{dt^2} = -\frac{\partial U(x;a)}{\partial x} - \gamma\frac{dx}{dt} + \xi(t)$$

$$\langle \xi(t)\xi(t')\rangle = 2\gamma k_B T\delta(t-t')$$

Heat: $$dQ \equiv \left(-\gamma\frac{dx}{dt} + \xi(t)\right)\circ dx =$$ **Force exerted from heat bath x displacement**

Then, 1 st Law of Thermodynamics holds for Langevin equations

$$d\left(\frac{p^2}{m} + U(x;a)\right) = dQ + \frac{\partial U}{\partial a}\circ da$$

$$dE = dQ + dW$$

$$E \equiv \frac{p^2}{m} + U(x;a)$$

$$dW \equiv \frac{\partial U}{\partial x}\circ da$$

How to derive

$$dQ = \left(\frac{dp}{dt} + \frac{\partial U}{\partial x} \right) \circ dx = \frac{dp}{dt} \circ \frac{p}{m} dt + dU - \frac{\partial U}{\partial x} \circ da$$

$$= d\left(\frac{p^2}{2m} \right) + dU - \frac{\partial U}{\partial x} \circ da$$

Separate conservative forces and non-conservative forces

$$d\left(\frac{p^2}{m} + U(x;a) \right) = dQ + \frac{\partial U}{\partial a} \circ da$$

$$dE = dQ + dW$$

1 st Law of Thermodynamics for Langevin Dynamics

$$E \equiv \frac{p^2}{m} + U(x;a) \qquad dW \equiv \frac{\partial U}{\partial x} \circ da$$

Ito Integral and Stratonovich Integral in Stochastic Differential Equations

Wiener process : B_t $\int_0^t \xi(s)ds = \sqrt{2\gamma k_B T}\left[B_t - B_0\right]$

Computing $\int f(s)dB_s$

Ito's definition

$$f(s)\cdot dB_s \equiv f(s)\left[B_{s+\Delta s} - B_s\right]$$

Stratonovich's defintion

$$f(s)\circ dB_s \equiv \frac{f(s+\Delta s)+f(s)}{2}\left[B_{s+\Delta s} - B_s\right]$$

Mesoscopic Nonequilibrium Systems

Some New Results

- **Jarzynski Equality** **(1997)**

  (Relation connecting Equilibrium and Nonequilibrium)

$$\overline{W} \geq \Delta F \quad : \quad \text{irreversible processes}$$

$$\exp(-\beta \Delta F) = \overline{\exp(-\beta W)}$$

Ex.： Unfolding of RNA

- **Fluctuation Theorem** **(1993-)**

$$\frac{P(\sigma)}{P(-\sigma)} = e^{\sigma\tau} \qquad \sigma : \text{entropy production rate}$$

（Probability of negative entropy production, Proof of the second law in thermodynamics）

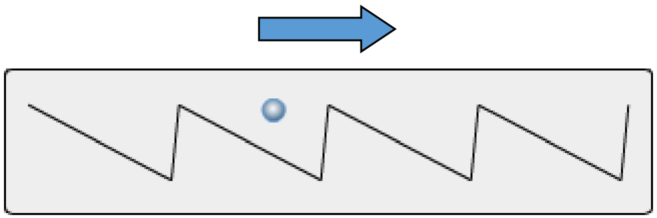
- **Harada-Sasa Equality** **(2005)**

FDT violation ＝ Irreversible Production of Heat

Ex： efficiency of molecular motors

Example: Small Particle Driven by External Forces

Langevin Simulation of Active Transport: ex. Brownian Ratchet



$U(x,t) = U(x,t)$

$\updownarrow$ flushing

$U(x,t) = 0$

Langevin Equation (overdamped case)

$$0 = -\gamma \dot{x} - \frac{\partial U(x,t)}{\partial x} + f(t) + \hat{\xi}(t)$$

$$\left\langle \hat{\xi}(t) \right\rangle = 0, \quad \left\langle \hat{\xi}(t)\hat{\xi}(0) \right\rangle = 2\gamma k_{\mathrm{B}} T \delta(t)$$

Energy Dissipation

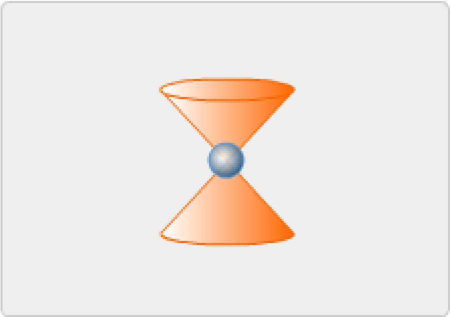
- Energy dissipation rate (energy flow from the Brownian particle to the environment) can be defined as

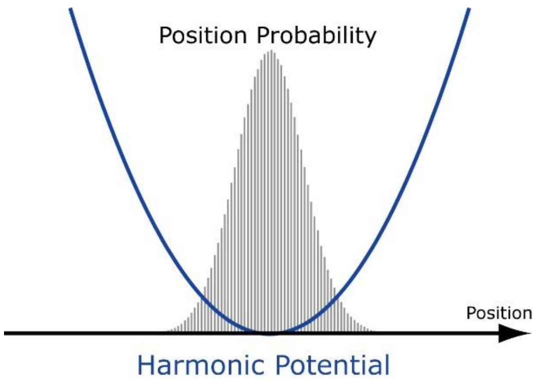
$$
\begin{aligned}
J(t)dt &\equiv \left[\gamma \dot{x}(t) - \hat{\xi}(t)\right] \circ dx(t) \\
&= \left[-\frac{\partial U(x,t)}{\partial x} + f(t)\right] \circ dx(t)
\end{aligned}
$$

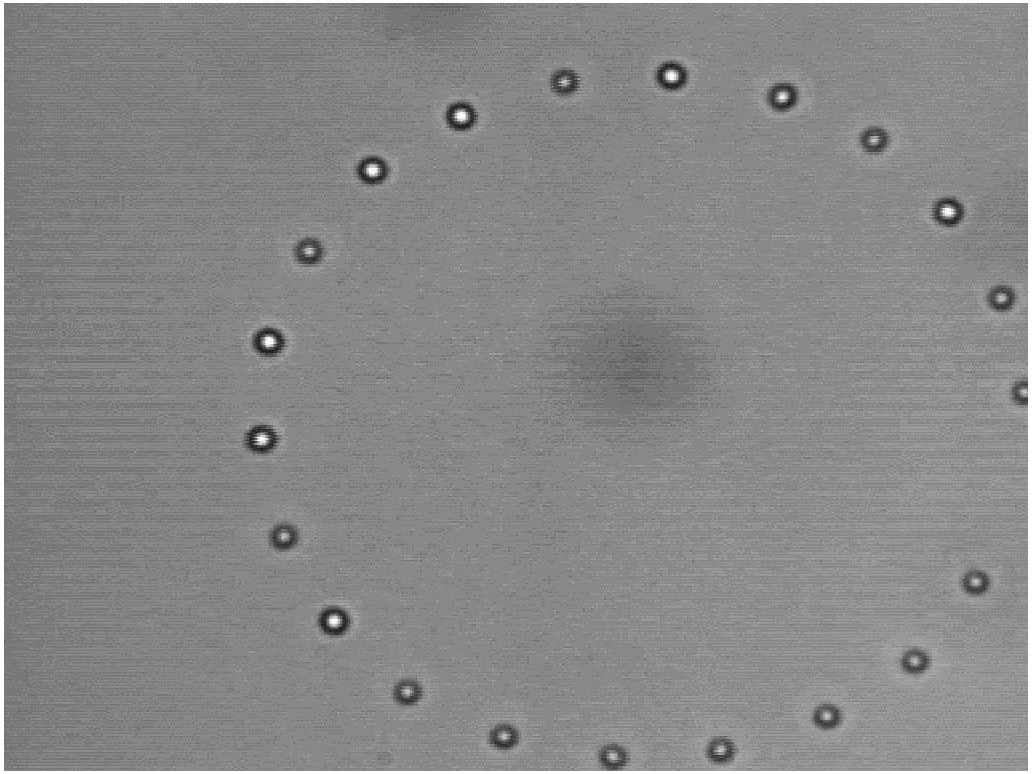
But difficult to measure.

according to Sekimoto(JPSJ, 1997).

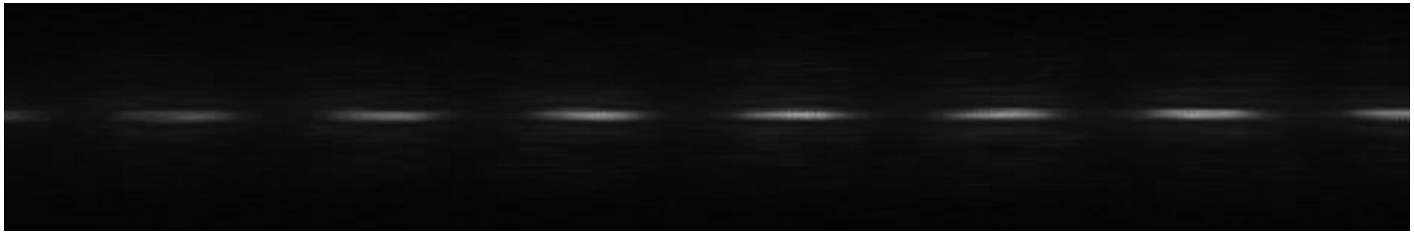
Optical Trapping (LASER Tweezers)



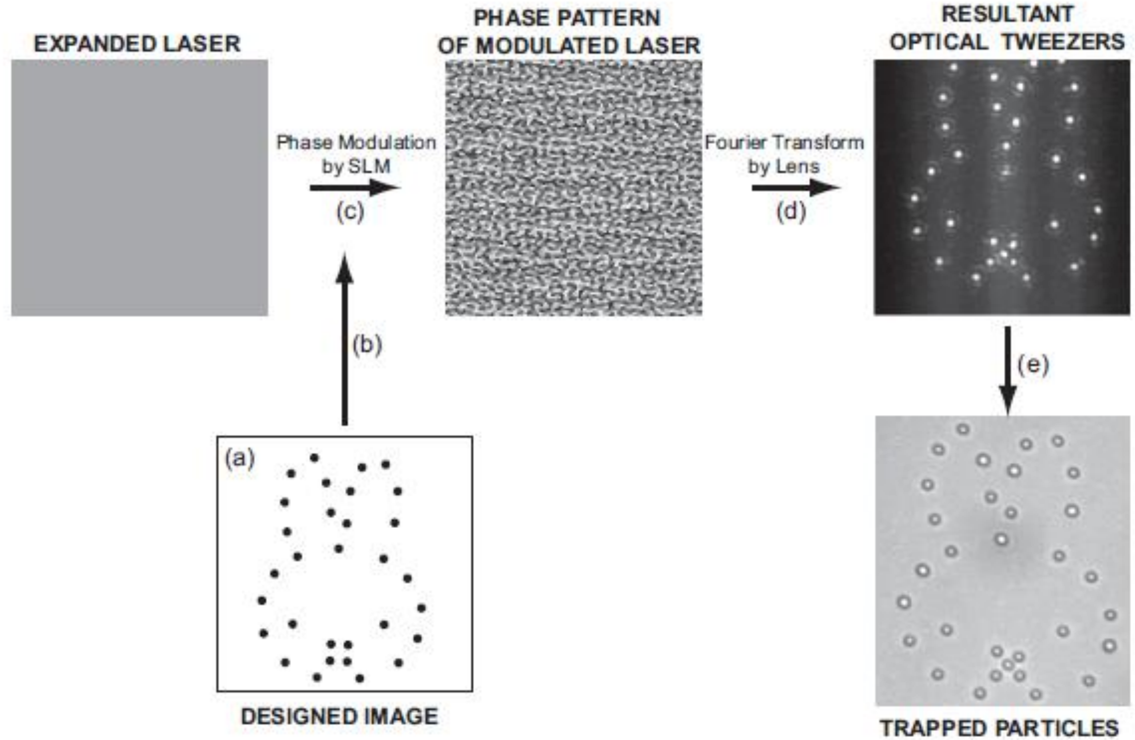


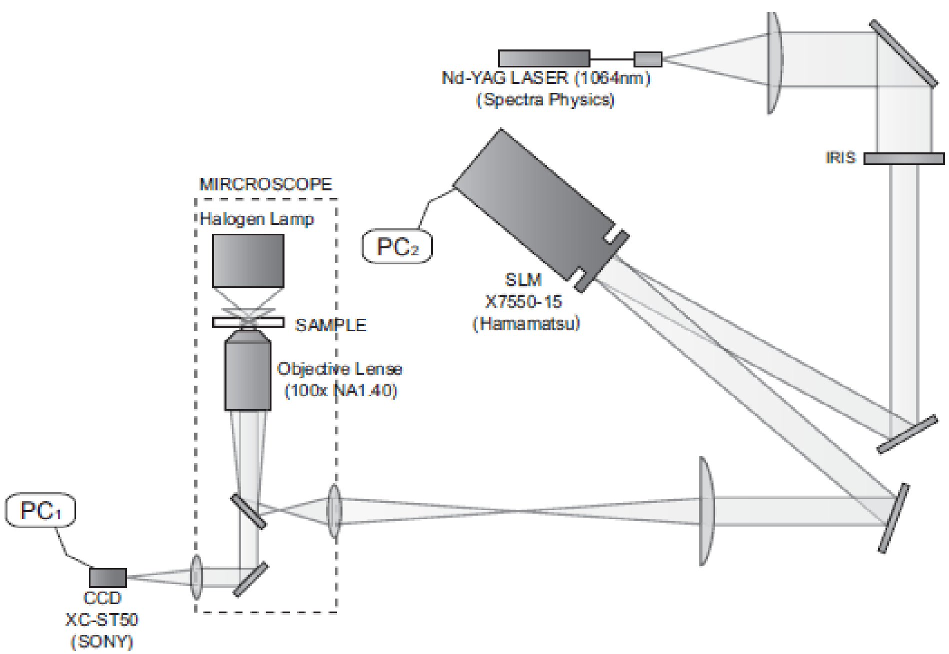


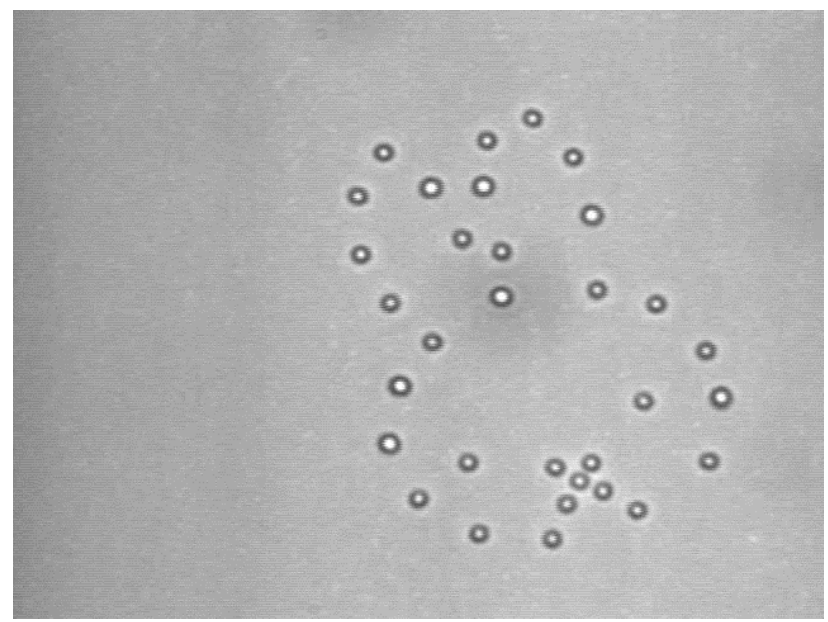
Toyabe & Sano (2006)



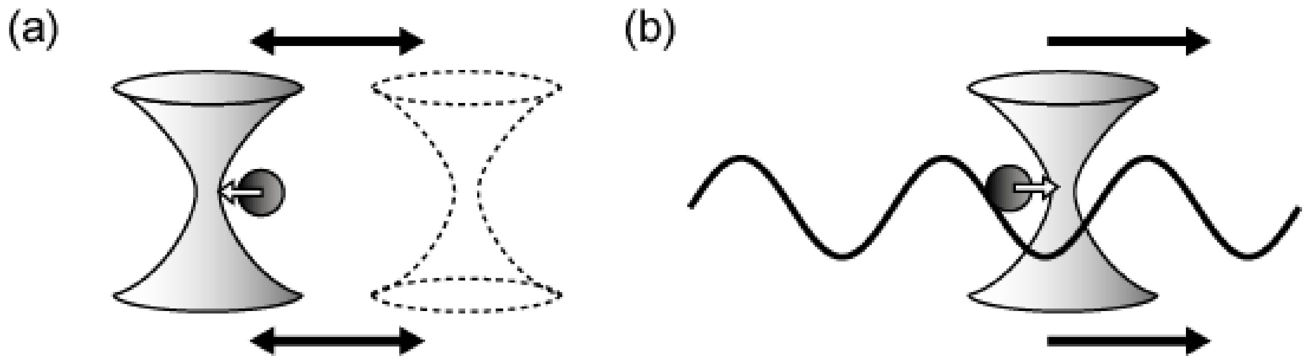
Multi-beam Laser Trap





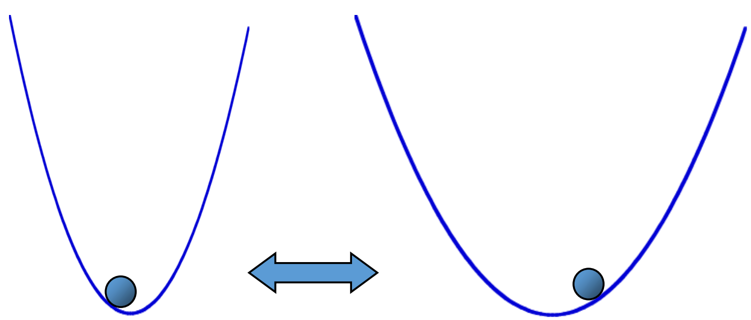


How to make Nonequilibrium State ?



Switching

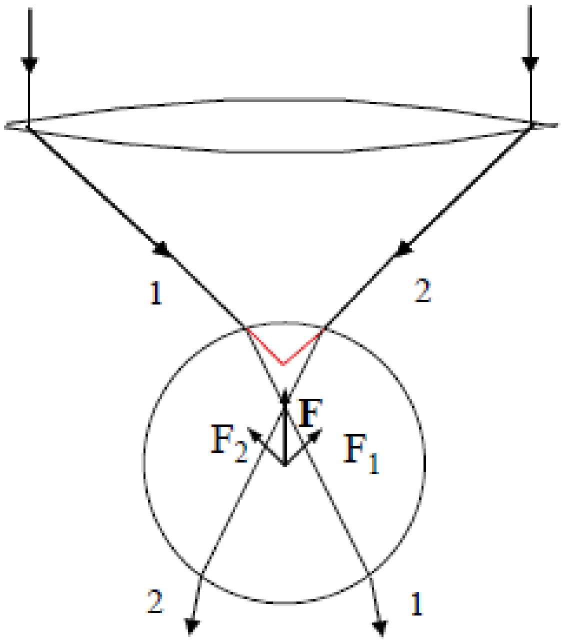
Surfing

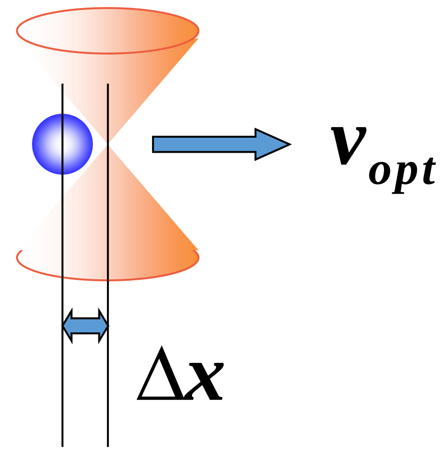


Swinging

Tool: Laser Tweezers and Colloid System

- It is possible to capture and manipulate small particles.
- Particles feel **harmonic potential** around the focal point of the LASER.





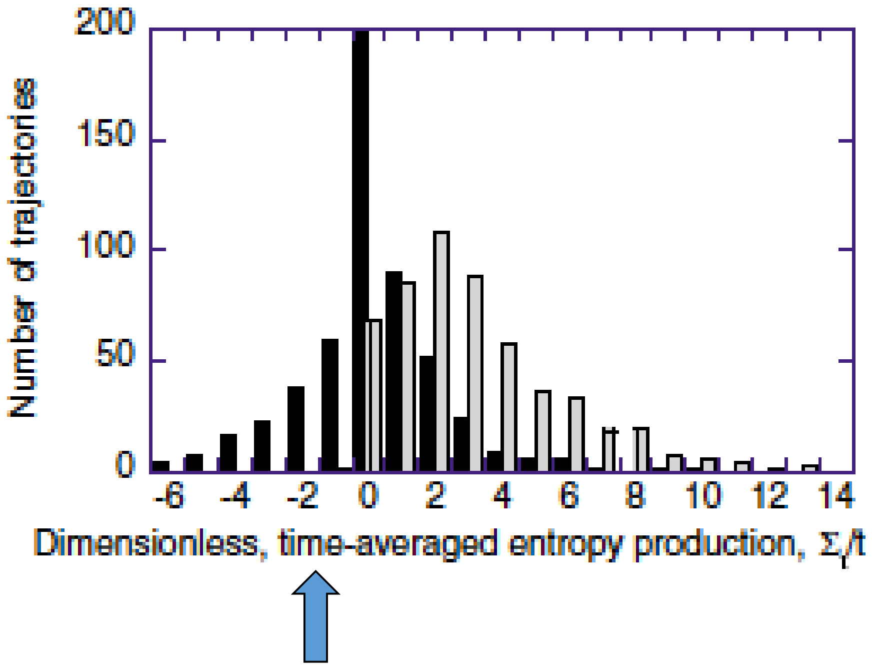
$$F_{opt} = k\Delta x = k(x_{opt}(t) - x(t))$$

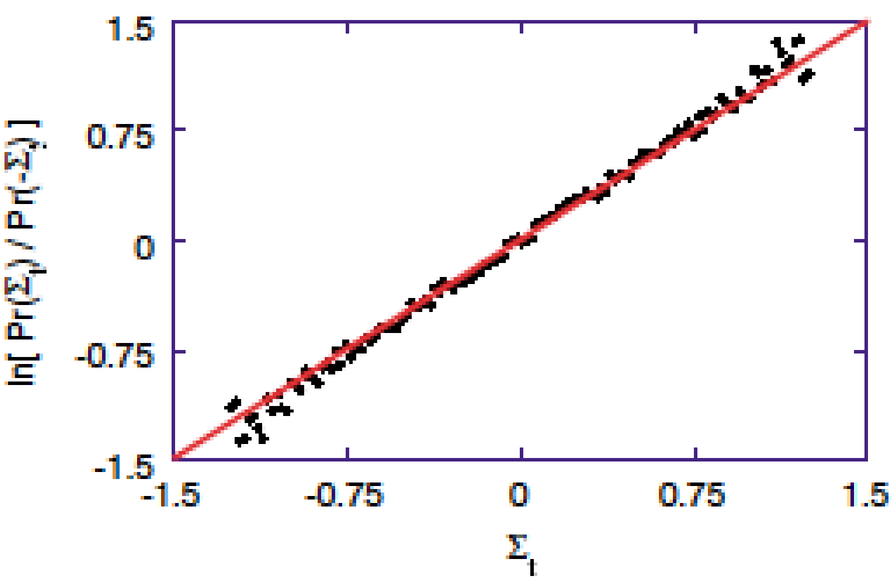
Energy Input : $$J(t) = F_{opt}(t)v_{opt}(t)$$

Entropy production : $$\sigma_t = \frac{1}{k_B T}\int_0^t F_{opt}(t)v_{opt}dt$$

Fluctuation Theorem (steady state)

$$\lim_{\tau\to\infty}\frac{1}{\tau}\log\frac{\Pi(\tau\sigma)}{\Pi(-\tau\sigma)}=\sigma \qquad \Pi: \text{distribution function}$$





Evans et al., PRL (2002)

Negative Entropy Production

Measuring Energy Dissipation in Small Systems

Harada-Sasa's relation, P.R.L. (2005):

$$\langle J \rangle = \gamma \int_{-\infty}^{\infty} \left[\tilde{C}(\omega) - 2k_{\mathrm{B}} T \tilde{R}'(\omega) \right] \frac{d\omega}{2\pi}$$

Heat = Degree of Violation of FDT

Equilibrium State

$$D = \mu k_B T \quad \Longleftrightarrow \quad \overline{C}(\omega) = 2k_B T \overline{R}(\omega)$$

Einstein's Relation (1905)

Correlation Function: $C(t) \equiv \langle \dot{x}(t) \dot{x}(0) \rangle$

Response Function: $\langle \dot{x}(t) \rangle_\epsilon = \epsilon \int_{-\infty}^{t} R(t-s) f^p(s) ds + O(\epsilon^2)$

$$C(\tau) = \frac{1}{2\pi} \int C(\omega) e^{i\omega\tau} d\omega$$

The Equality is generalized for more complex systems

- Time independent driving force with U(x)
- Flushing potential
- Periodically switching of potential : U(x,t+T)=U(x,t)
- Many particles (Colloidal suspension)

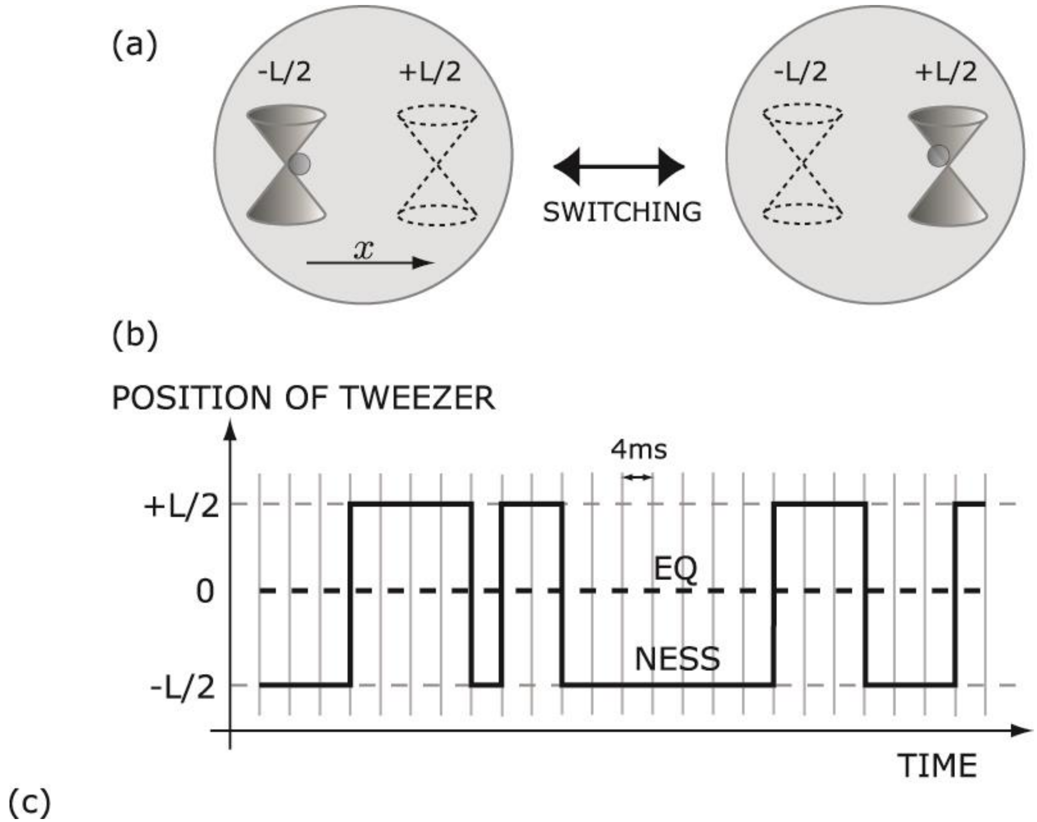
$$\gamma \dot{x}_i(t) = F_i(\Gamma(t)) + \xi_i(t) + \varepsilon f_i^{\mathrm{p}}(t),$$

$$F_i(\Gamma) = \sum_{\mu=1}^{N} f \delta_{i,3\mu-2} - \partial_{x_i} \sum_{\mu=1}^{N} U(\boldsymbol{r}_\mu) - \partial_{x_i} \sum_{\mu,\nu=1}^{N} U_{\mu\nu}^{\mathrm{int}}(|\boldsymbol{r}_\mu - \boldsymbol{r}_\nu|)/2$$

$$\langle \xi_i(t)\xi_j(s)\rangle_0 = 2\gamma T \delta_{ij}\delta(t-s).$$

$$\langle J\rangle_0 = \sum_{i=1}^{3N} \gamma \left\{ \langle \dot{x}_i \rangle_0^2 + \int_{-\infty}^{\infty} [\tilde{C}_{ii}(\omega) - 2T\tilde{R}'_{ii}(\omega)] \frac{d\omega}{2\pi} \right\},$$

How to make Nonequilibrium Steady State (NESS)?

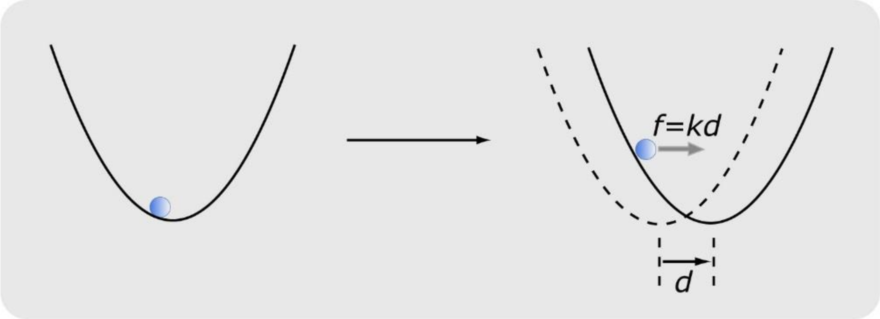


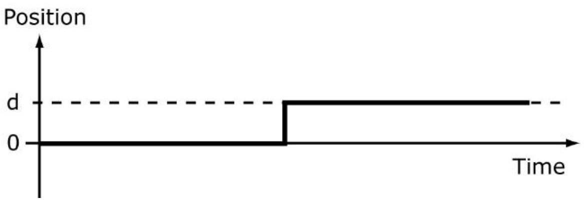
Equilibrium Case

Check of FDT(Fluctuation Dissipation Theorem)

FDT

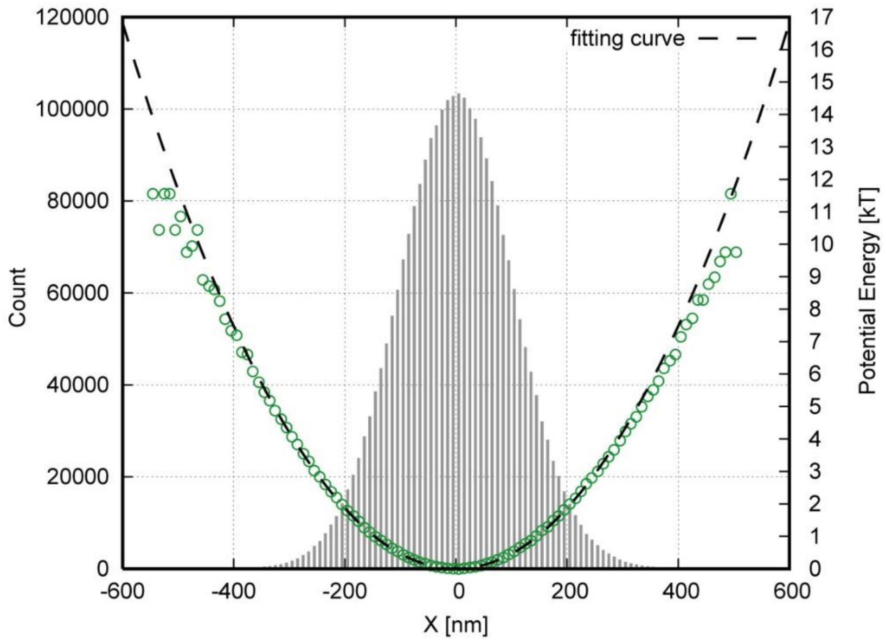
$$\tilde{C}(\omega) = 2k_{\mathrm{B}}T\tilde{R}'(\omega)$$





$$\begin{aligned}\gamma\dot{x} &= -k(x-d) + \hat{\xi}(t) \\ &= -kx + kd + \hat{\xi}(t)\end{aligned}$$

Shape of the potential



2,654,127 POINTS

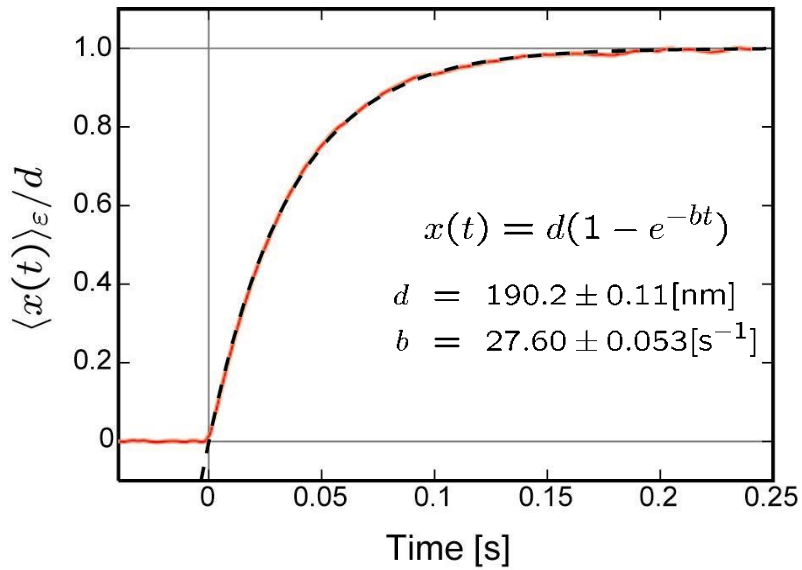
$$p(x) \propto \exp\left(-\frac{U(x)}{k_B T}\right)$$

$$U(x) = \frac{1}{2}kx^2$$

$$\begin{aligned} k &= 117.5 \pm 0.48[k_B T/\mu m^2] \\ &= 0.4870 \pm 0.0020[pN/\mu m] \end{aligned}$$

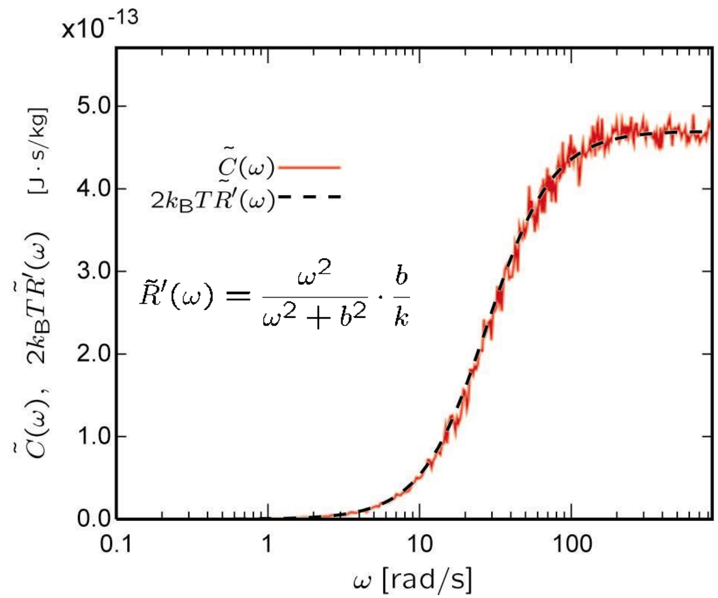
Equilibrium State (Check of Fluctuation-Dissipation Theorem)

7854 RUNS



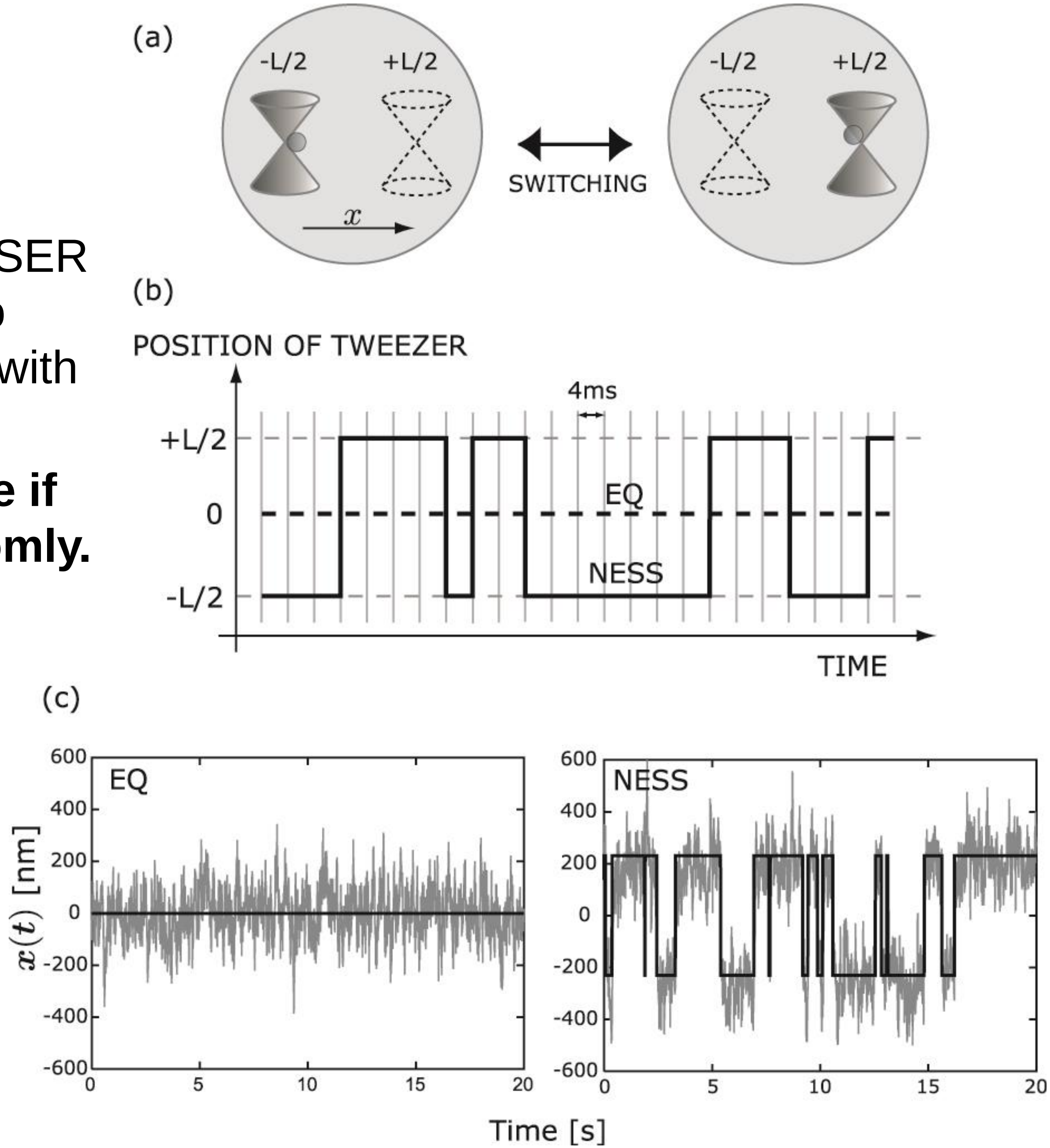
$$\gamma = \frac{k}{b} = 1.763 \times 10^{-8} [\mathrm{kg/s}]$$

52 RUNS x 32767 POINTS

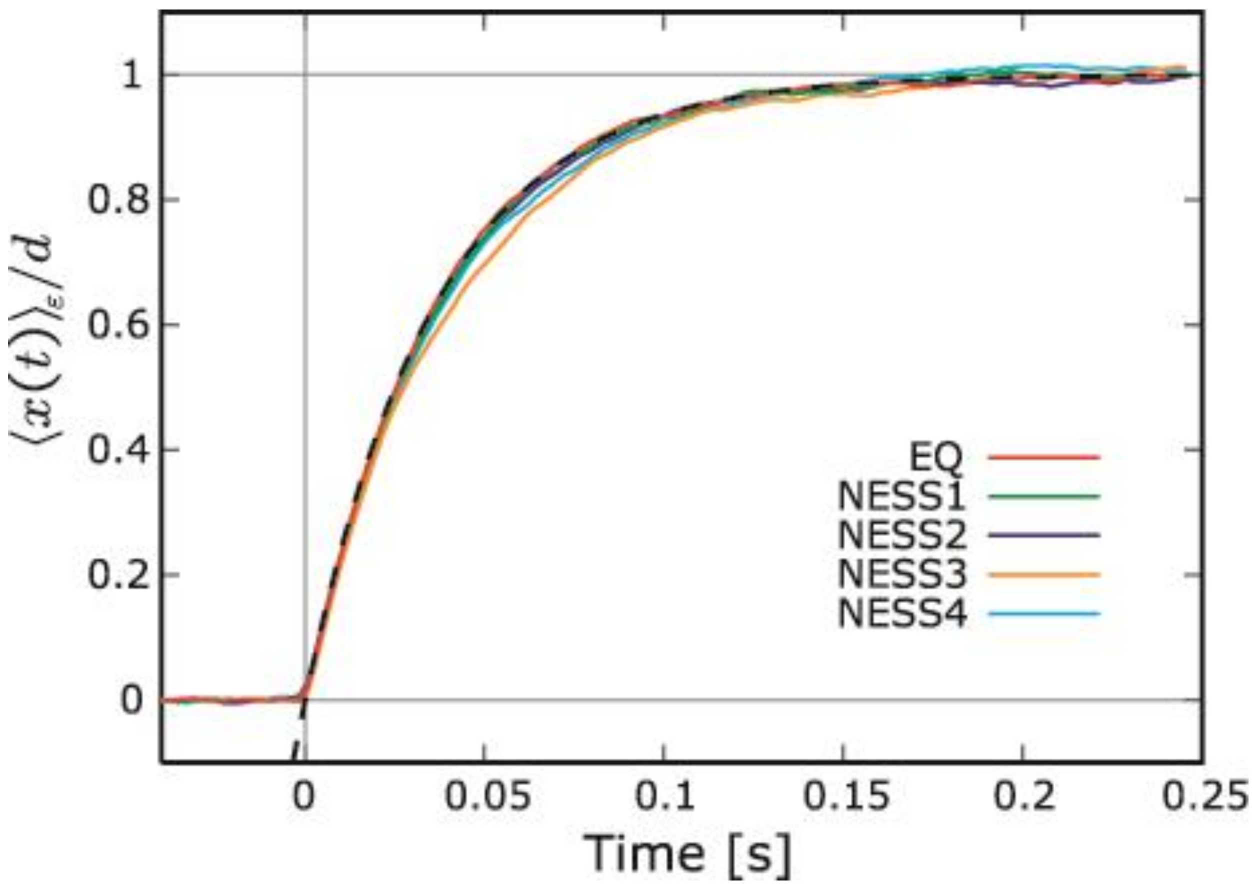


Non-Equilibrium Case

- Flushing: Switch LASER position between two positions temporally with a **poisson process**.
- **Every 10ms, decide if switch or not randomly.**



Measurement of Response Functions



EQ: Equilibrium State

switching rate

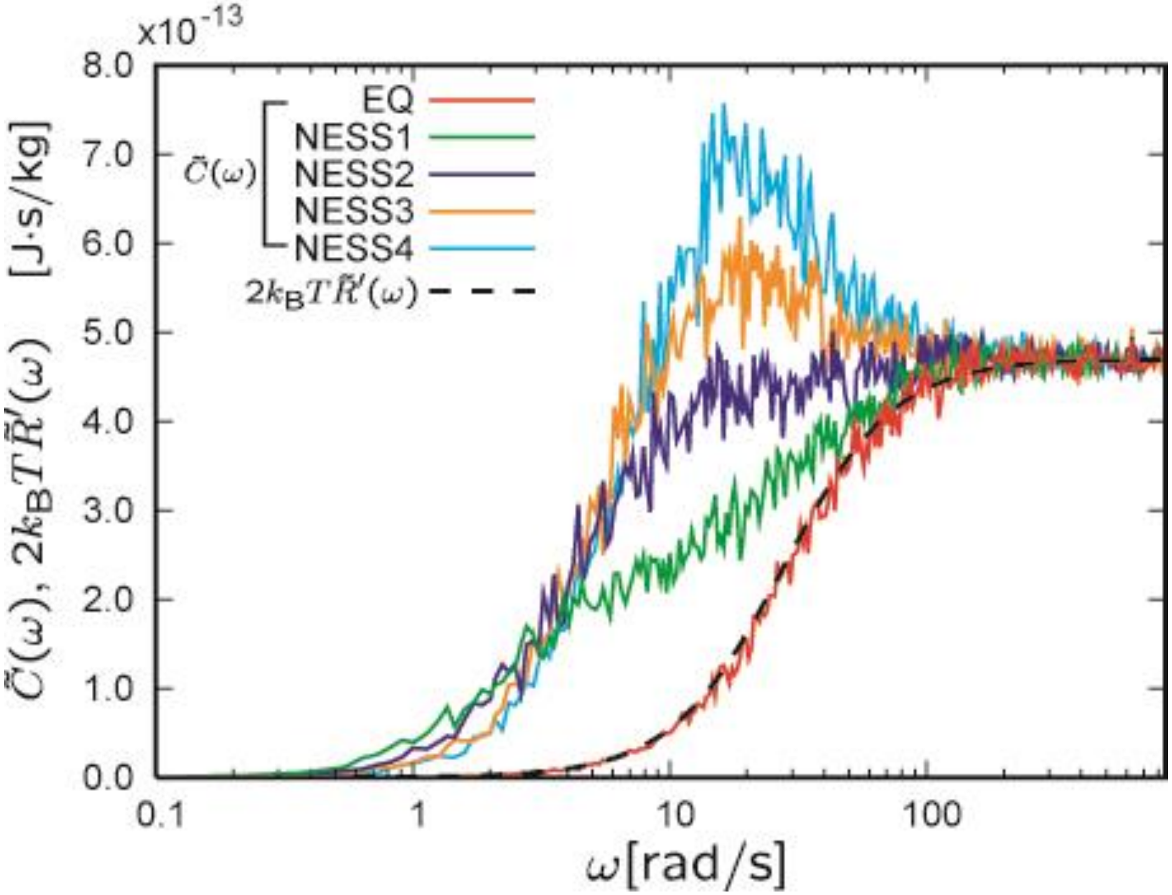
NESS1: 1 sec^{-1}

NESS2: 2 sec^{-1}

NESS3: 3 sec^{-1}

NESS4: 4 sec^{-1}

Correlation Functions & Responses



EQ: Equilibrium State

switching rate

NESS1: 1 sec^{-1}

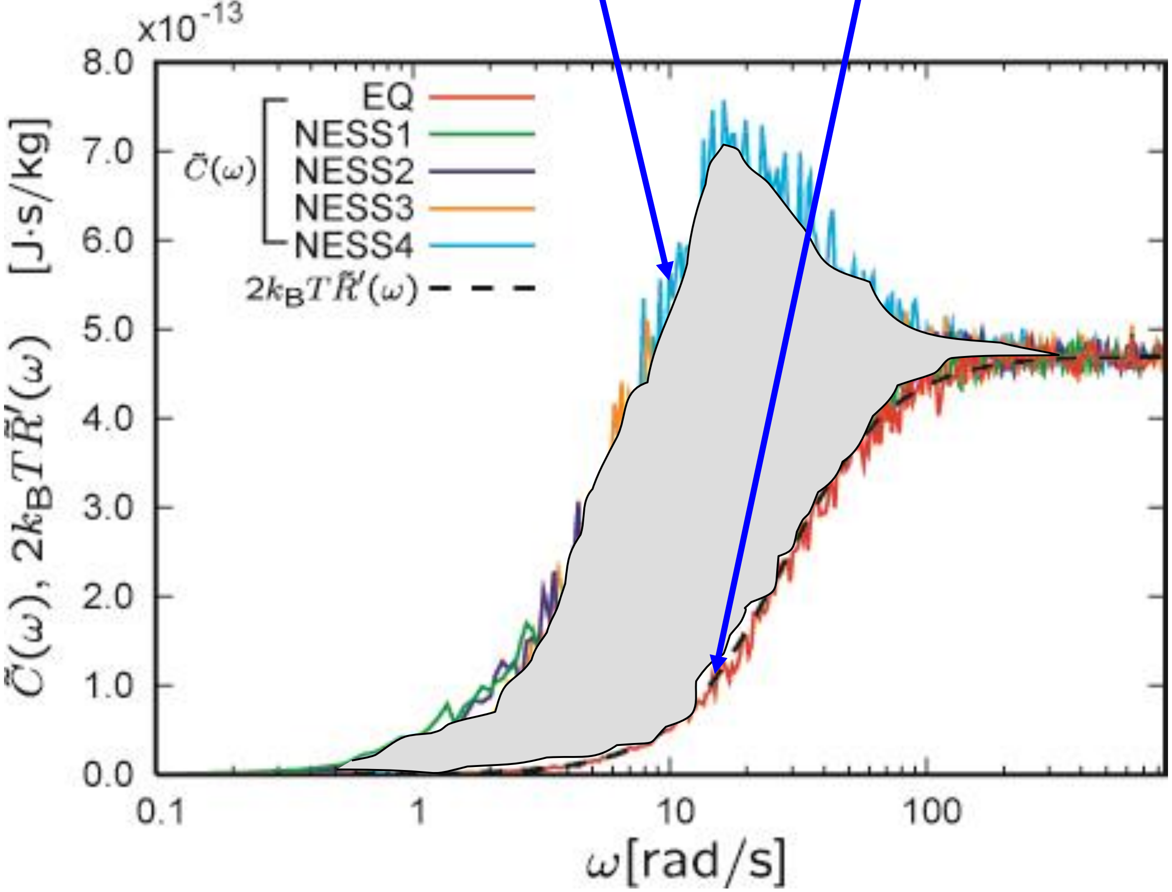
NESS2: 2 sec^{-1}

NESS3: 3 sec^{-1}

NESS4: 4 sec^{-1}

Correlation Functions & Responses

$$\langle J \rangle = \gamma \int_{-\infty}^{\infty} \left[\tilde{C}(\omega) - 2k_B T \tilde{R}'(\omega) \right] \frac{d\omega}{2\pi}$$



EQ: Equilibrium State

switching rate

NESS1: 1 sec^{-1}

NESS2: 2 sec^{-1}

NESS3: 3 sec^{-1}

NESS4: 4 sec^{-1}

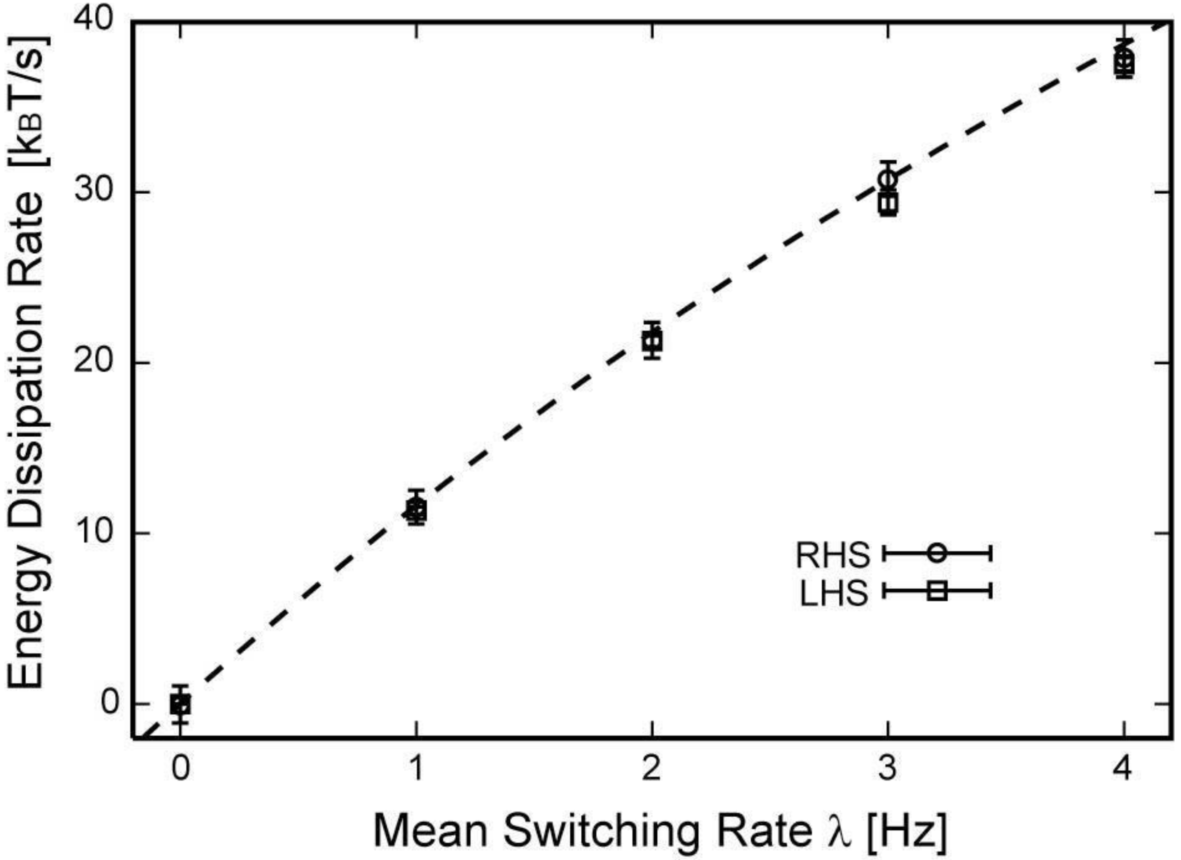
$$C(\omega) = v(\omega)v^*(\omega) = \omega^2 x(\omega) x^*(\omega)$$

Verification of Harada-Sasa Equality

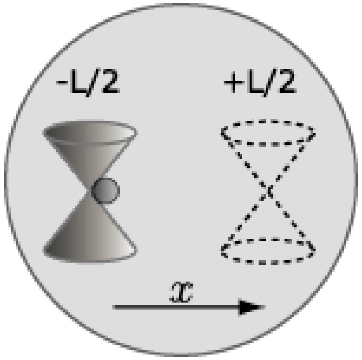
$$\langle J \rangle = \gamma \int_{-\infty}^{\infty} \left[\tilde{C}(\omega) - 2k_B T \tilde{R}'(\omega) \right] \frac{d\omega}{2\pi}$$

LHS can be measured in this system

$$\begin{aligned} \langle J \rangle &= \langle F(t) \circ \dot{x}(t) \rangle \\ &= -k \langle [x(t) - x_0(t)] \circ \dot{x}(t) \rangle \end{aligned}$$



More general case (Memory effect)



Colloid in polymer solution: viscosity has memory effect → generalized Langevin eq.

$$\int_{-\infty}^{t} \gamma(t-s)\dot{x}(s)ds = F(x(t),t) + \xi(t)$$

$$\tilde{I}(\omega) = \tilde{\Gamma}'(\omega)\left[\tilde{C}(\omega) - 2kT\tilde{R}'(\omega)\right]$$

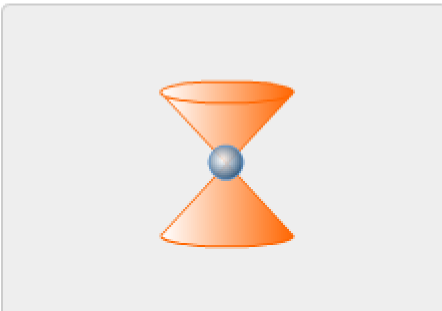
$$I(t) \equiv \frac{\langle F(x(t),t) \circ v(0)\rangle_0 + \langle F(x(0),0) \circ v(t)\rangle_0}{2}$$

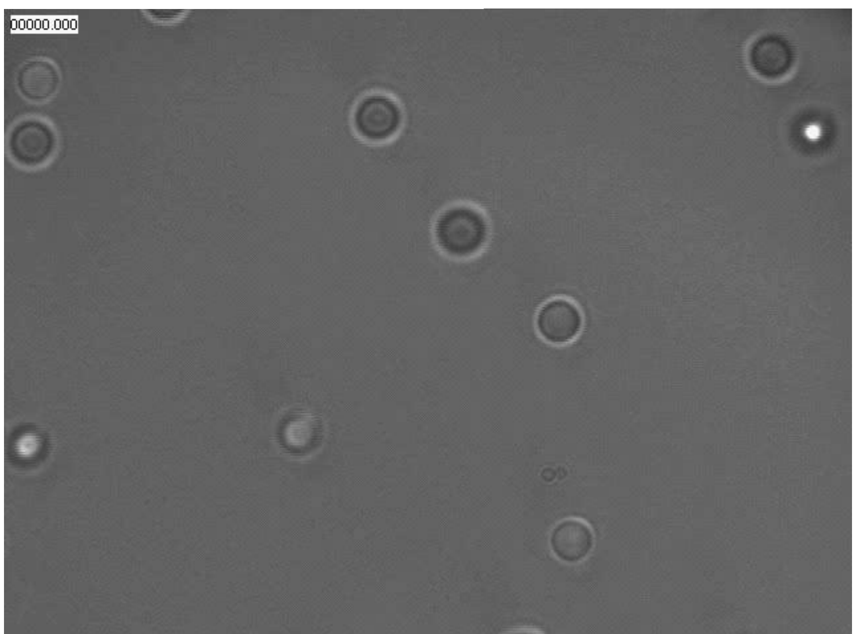
Ohkuma & T. Ohta (2006)

Narayan, Deutche PRE (2006)

Micro-Rheology

Measure viscosity from fluctuations





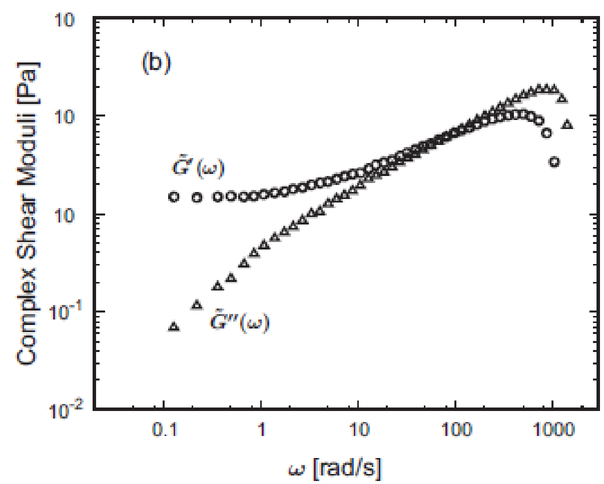
FDT:

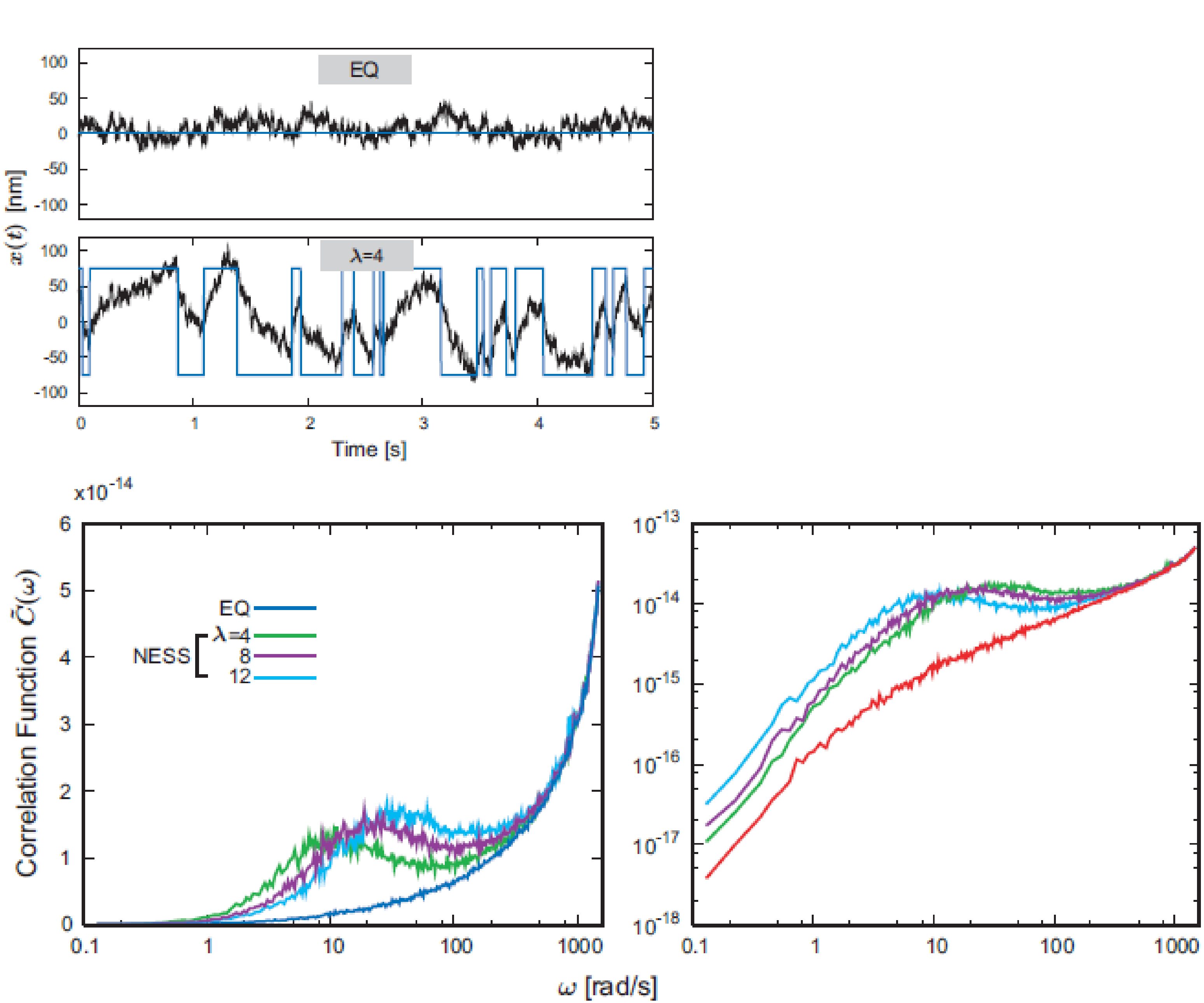
$$\tilde{C}_x(\omega) \equiv \langle |\tilde{x}(\omega)|^2 \rangle = \frac{2k_B T \tilde{\alpha}''(\omega)}{\omega},$$

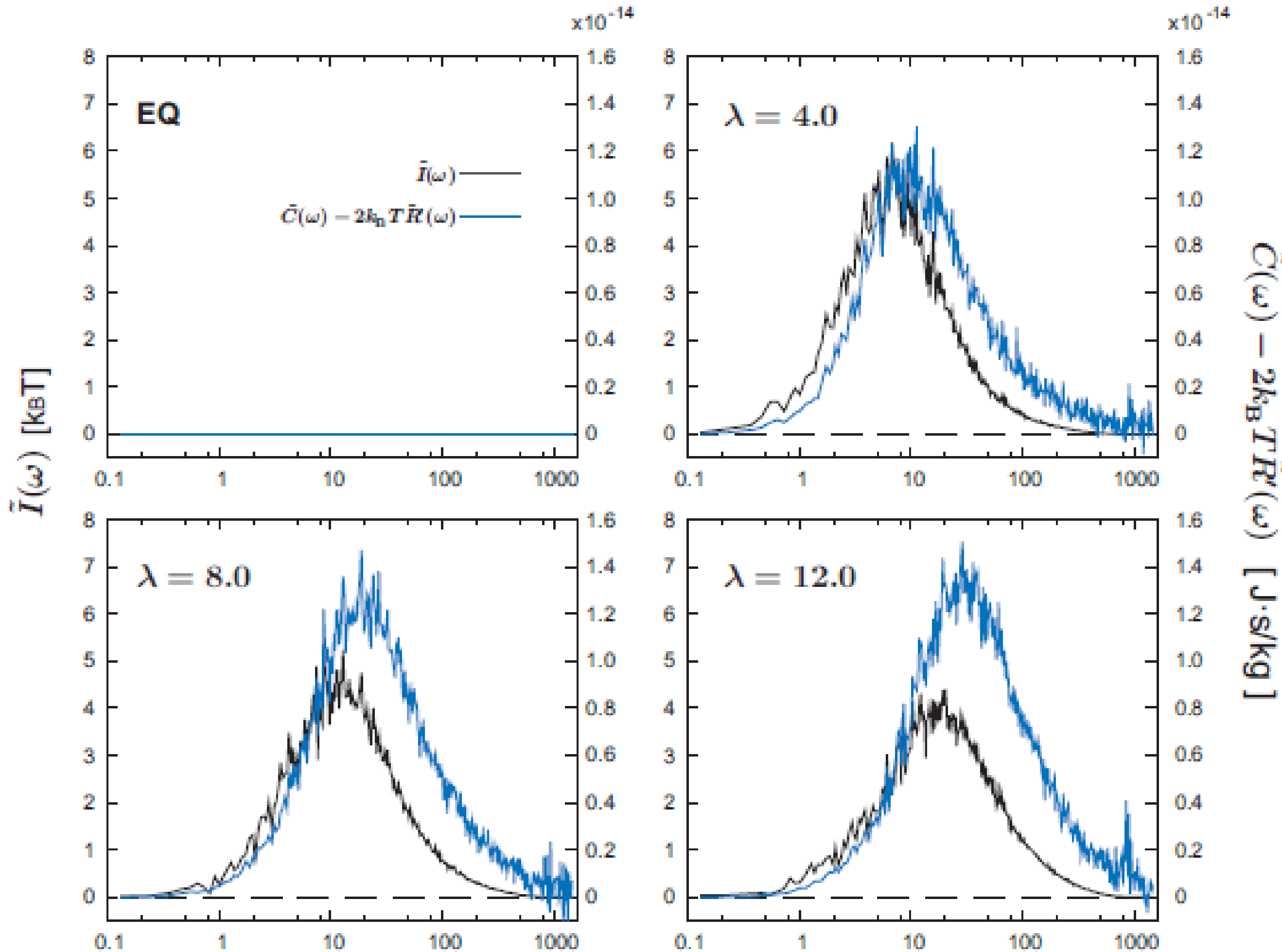
$$\tilde{x}(\omega) = \tilde{\alpha}(\omega)\tilde{f}(\omega).$$

$$\tilde{\alpha}'(\omega) = \frac{2}{\pi} P \int_0^\infty \frac{\zeta \tilde{\alpha}''(\zeta)}{\zeta^2 - \omega^2} d\zeta$$

$$\tilde{G}(\omega) = \frac{1}{6\pi a \alpha(\omega)},$$

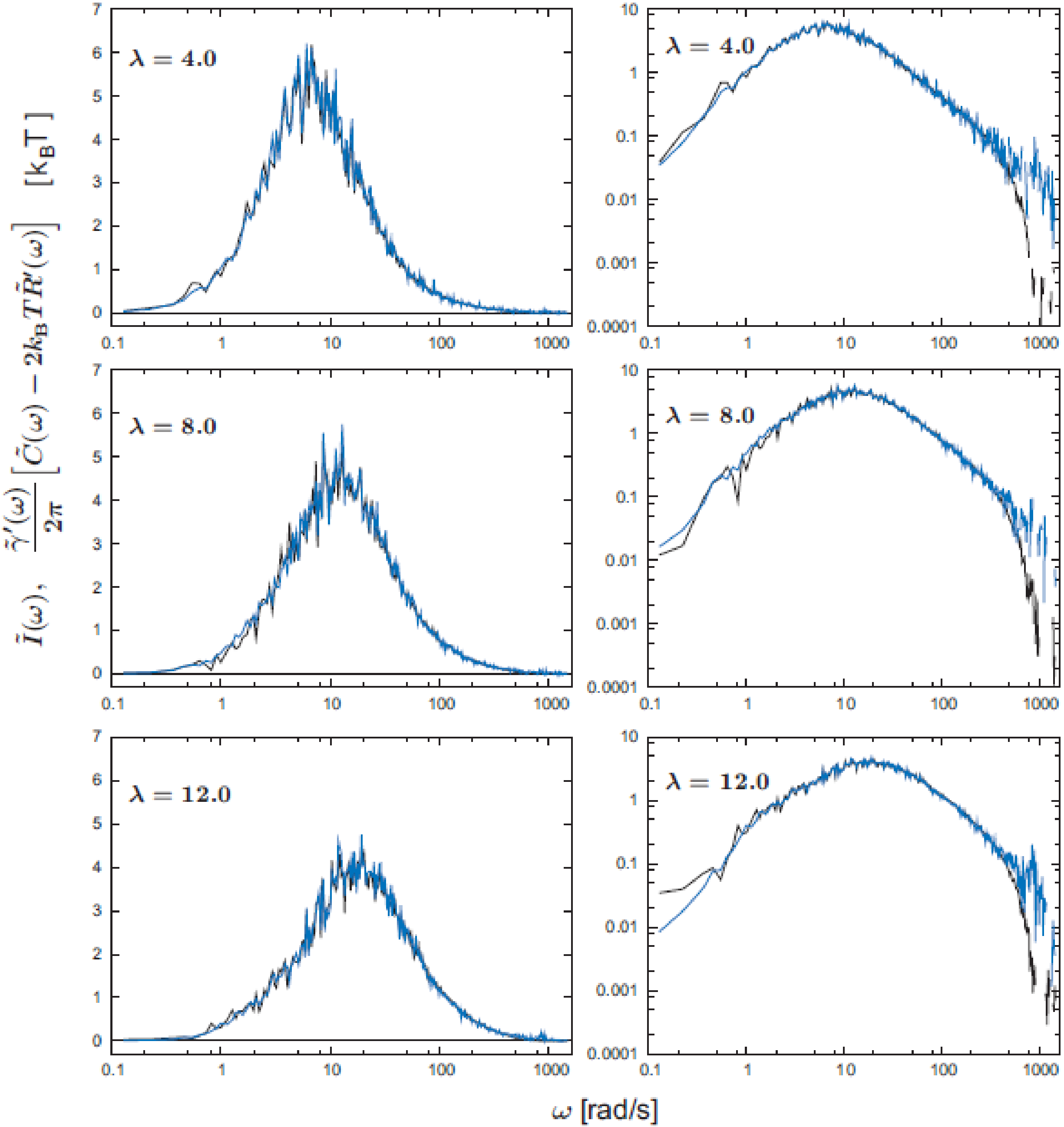


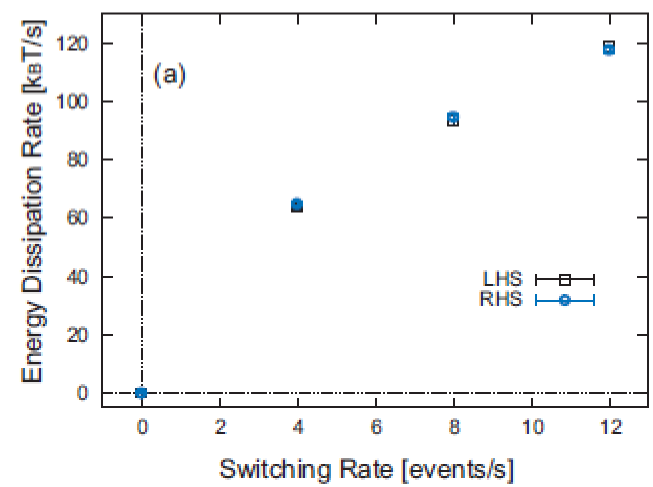

EQ
λ=4
x(t) [nm]
Time [s]
x10^-14
Correlation Function $\tilde{C}(\omega)$
EQ
NESS
λ=4
8
12
ω [rad/s]



$$\tilde{I}(\omega) = \gamma \left[\tilde{C}(\omega) - 2k_{\mathrm{B}} T \tilde{R}'(\omega) \right]$$

$$\tilde{I}(\omega) = \tilde{\Gamma}'(\omega)\left[\tilde{C}(\omega) - 2kT\tilde{R}'(\omega)\right]$$





Simplified proof !!

$$J(t)dt = (\gamma v(t) - \xi(t)) \circ dx(t)$$
$$J(t) = \gamma v(t)^2 - \xi(t) \circ v(t)$$

1st term:

$$< v(t)^2 > = v_s^2 + < (v(t) - v_s)^2 >_0$$
$$= v_s^2 + \int_{-\infty}^{\infty} C(\omega) \frac{d\omega}{2\pi}$$

response:

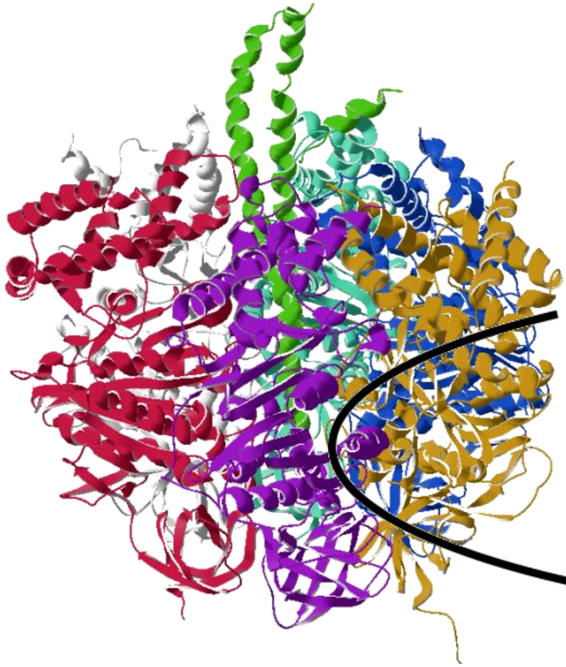
$$v(t) = \int_{-\infty}^{t} R(t-s)\xi(s)ds$$
$$V(\omega) = \int_{-\infty}^{\infty} R(\omega)\Sigma(\omega)e^{i\omega t} \frac{d\omega}{2\pi}$$

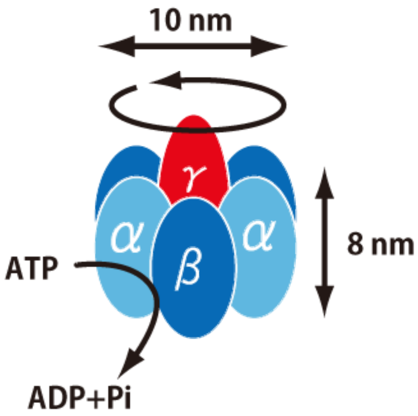
2nd term:

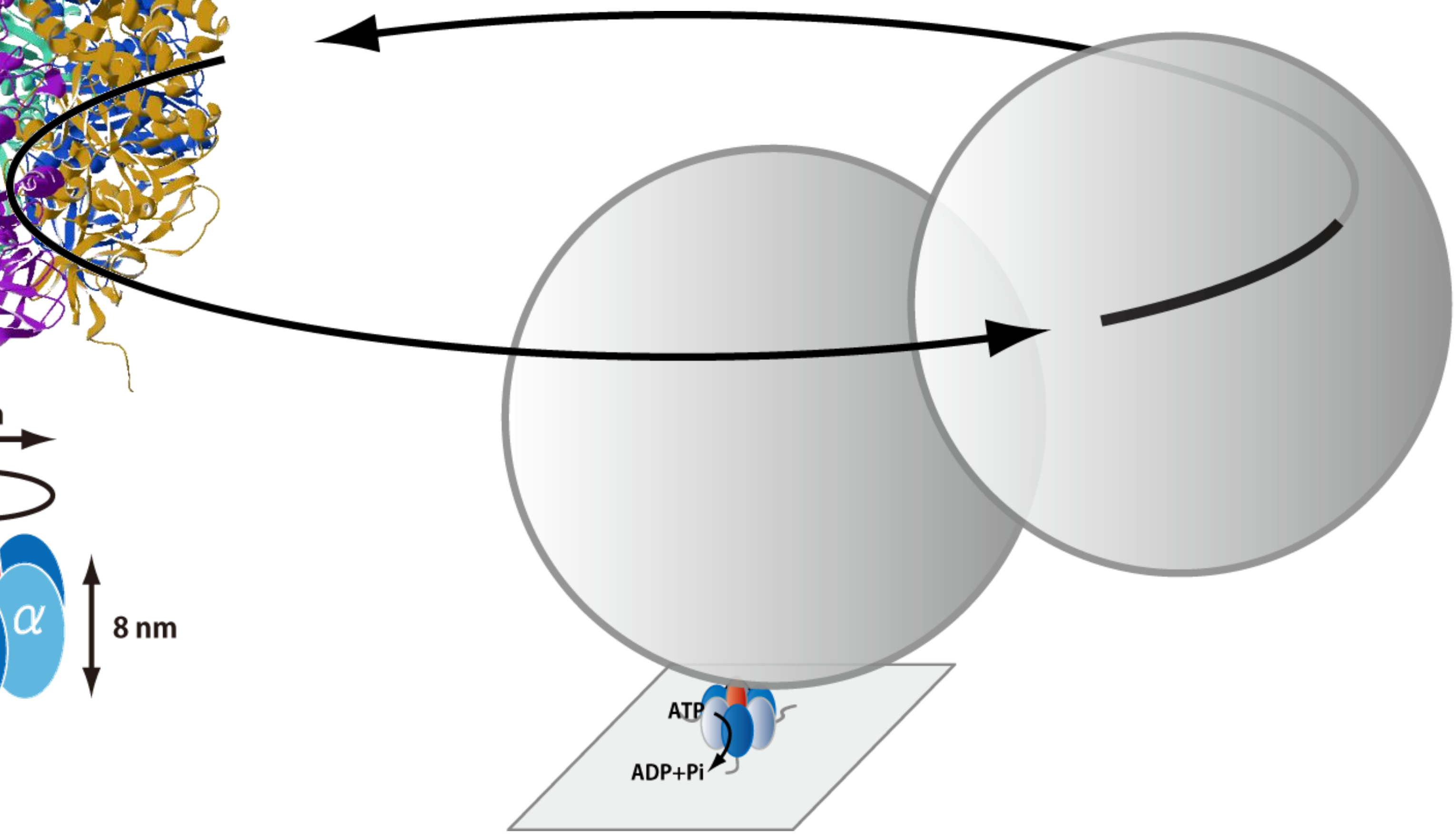
$$< \xi(t)v(t) > = \int_{-\infty}^{\infty} \xi(t)dt \int_{-\infty}^{\infty} R(\omega)\Xi(\omega)e^{i\omega t} \frac{d\omega}{2\pi}$$
$$= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} R(\omega)\Xi(\omega) \int_{-\infty}^{\infty} \xi(t)e^{i\omega t}dt$$
$$= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} R(\omega)\Xi(\omega)\Xi^*(\omega)$$
$$= \int_{-\infty}^{\infty} 2k_B T\gamma R(\omega) \frac{d\omega}{2\pi}$$

Molecular Rotary Motor > F_1-ATPase

- Abrahams *et al.*, *Nature*(1994)
- Noji, Yasuda *et al.*, *Nature*(1997)
- Yasuda, Noji *et al.*, *Cell*(1998)



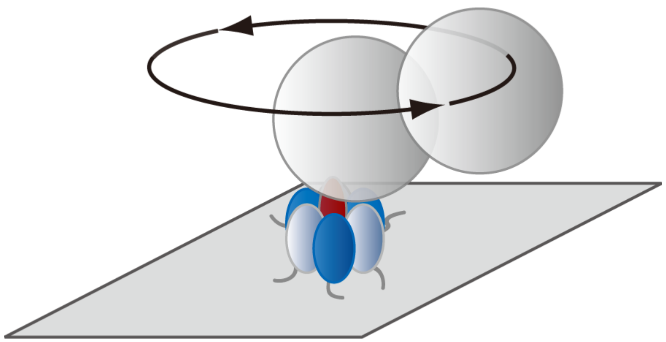


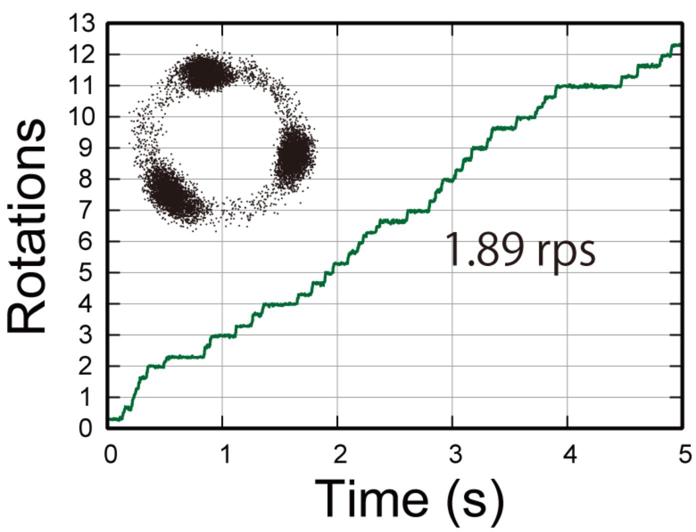


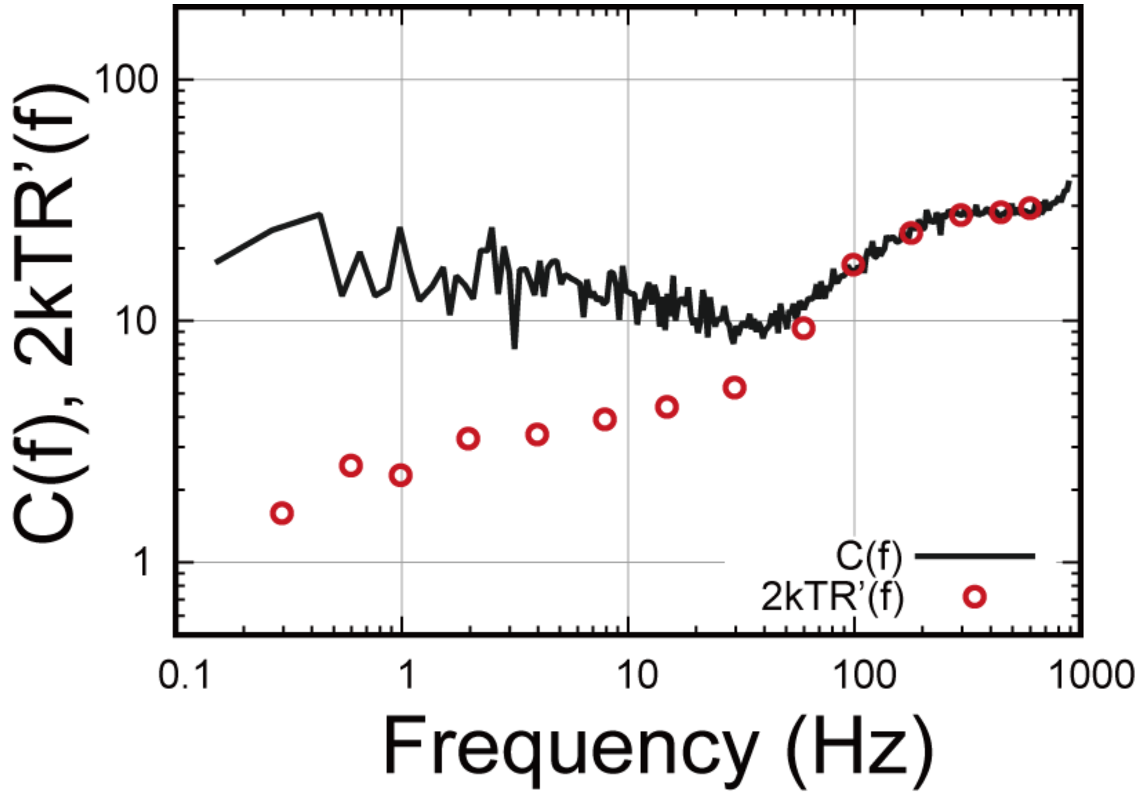
S. Toyabe et al. Phys. Rev. Lett. 104, 198103 (2010)

Results > Violation of FDT

- 0.4 uM ATP, 0.4 uM ADP, 1 mM Pi





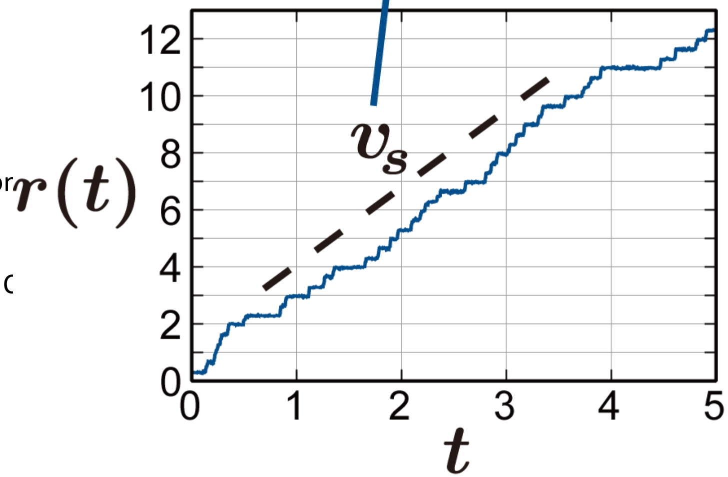


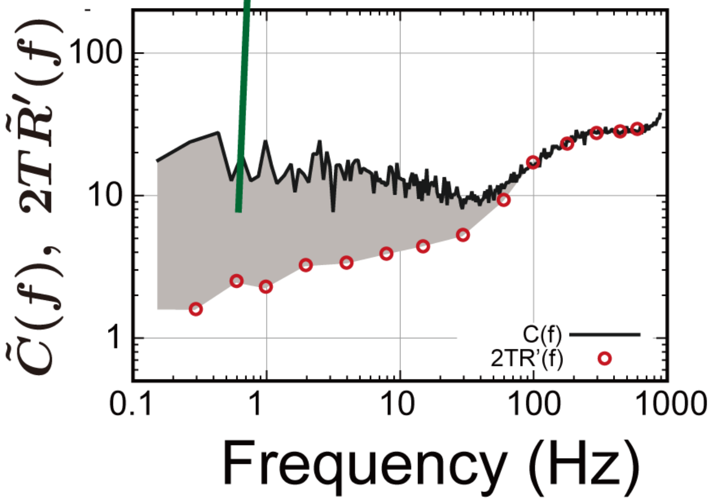
S. Toyabe et al. Phys. Rev. Lett. 104, 198103 (2010)

Results > Harada-Sasa equality

$$\langle J \rangle = \gamma v_s^2 + \gamma \int_{-\infty}^{\infty} df \left[\tilde{C}(f) - 2T\tilde{R}'(f) \right]$$

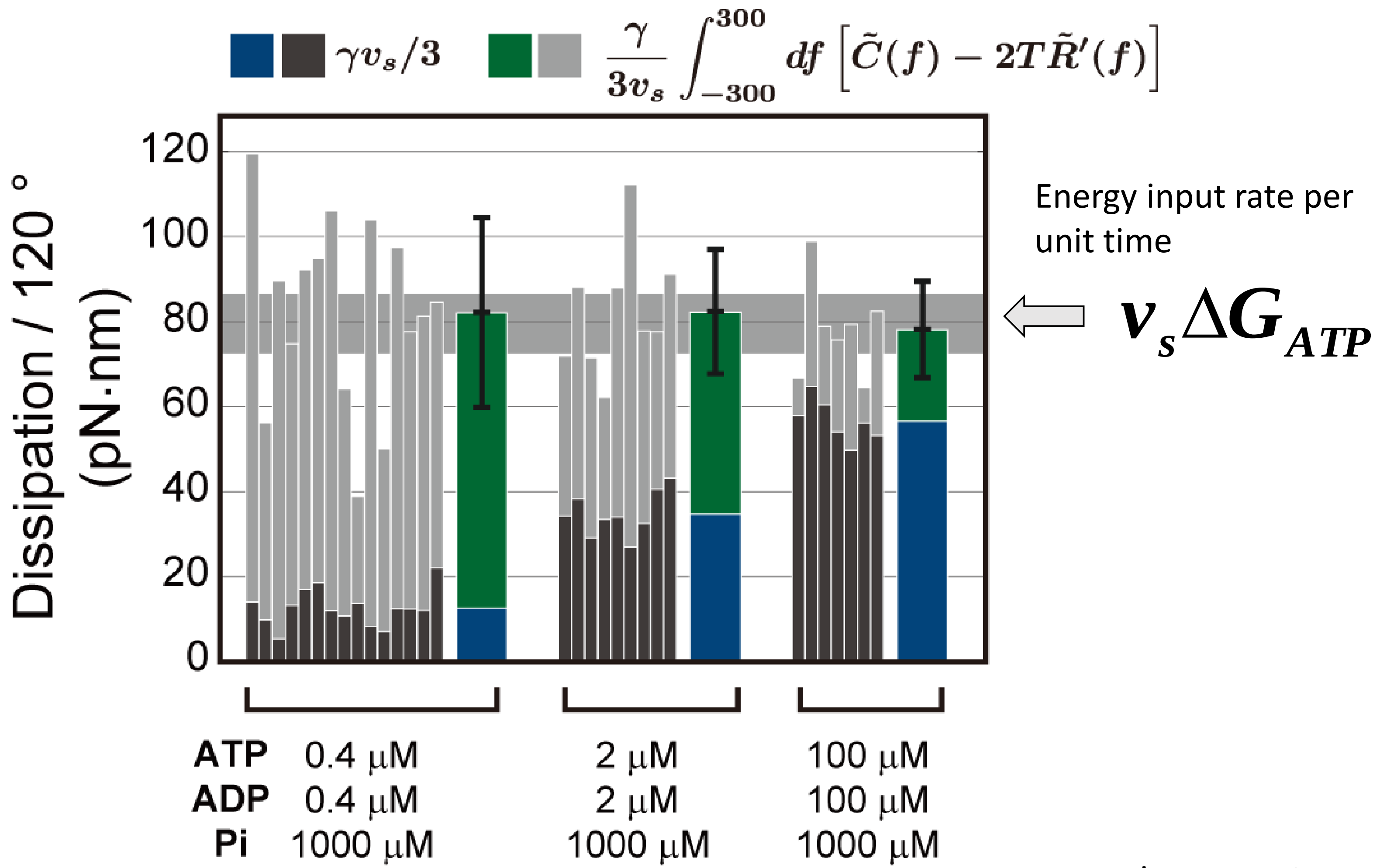
- Integration
- Frictional (





$$\tilde{C}(f \to \infty) \to \frac{2\gamma}{T}$$

Results > Heat Dissipation



Error bars are S.D.

S. Toyabe et al. Phys. Rev. Lett. 104, 198103 (2010)

From Brownian to Driven and Active Dynamics of Colloids: Energetics and Fluctuations Part II

Masaki Sano*
Shoichi Toyabe[1]
Hong-ren Jiang[2]
Ryo Suzuki*[3]
*The University of Tokyo
[1]Ludwig Maximilians University Munich
[2]National Taiwan University
[3]Techinical University of Munich

Outline of Part I

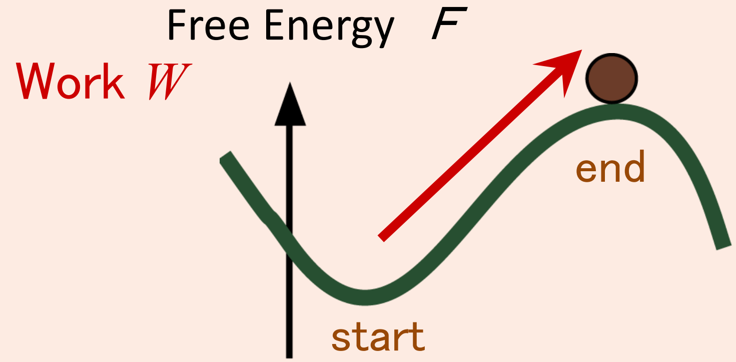
- Stochastic Thermodynamics was introduced

- Degree of the violation FDT relation is equal to heat flux to the environment in Langevin systems. Such small heat flux can be experimentally evaluated by using a new theory.

Jarzynski Equality

Importance of fluctuations

2 nd law of thermodynamics and FDT can be derived from JE



2 nd law of Thermodynamics

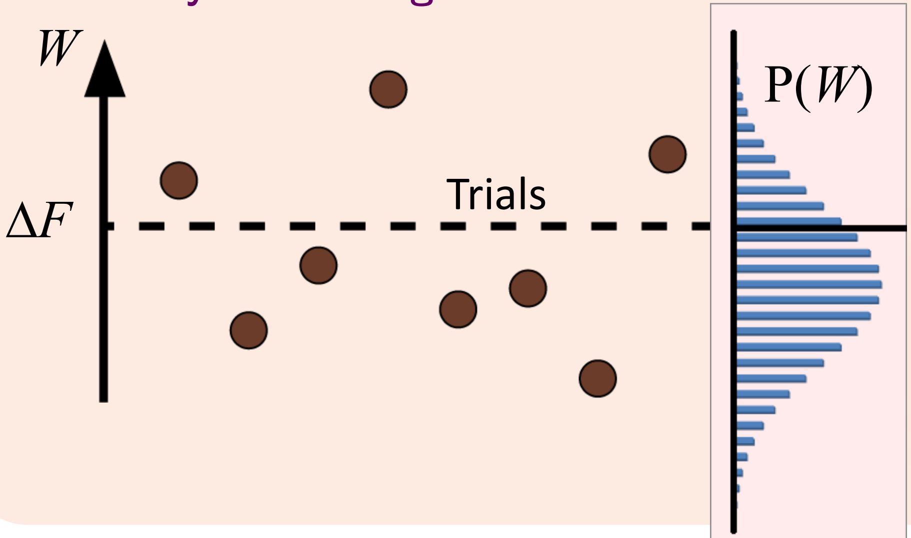
$$W \geq \Delta F = F_2 - F_1$$

Maximum work

W :Work exerted to the system

ΔF :Free energy gain of the system

Small system large fluctuation



Jarzynski equality (1997)

$$\overline{\exp[W/kT]} = \exp[\Delta F/kT]$$

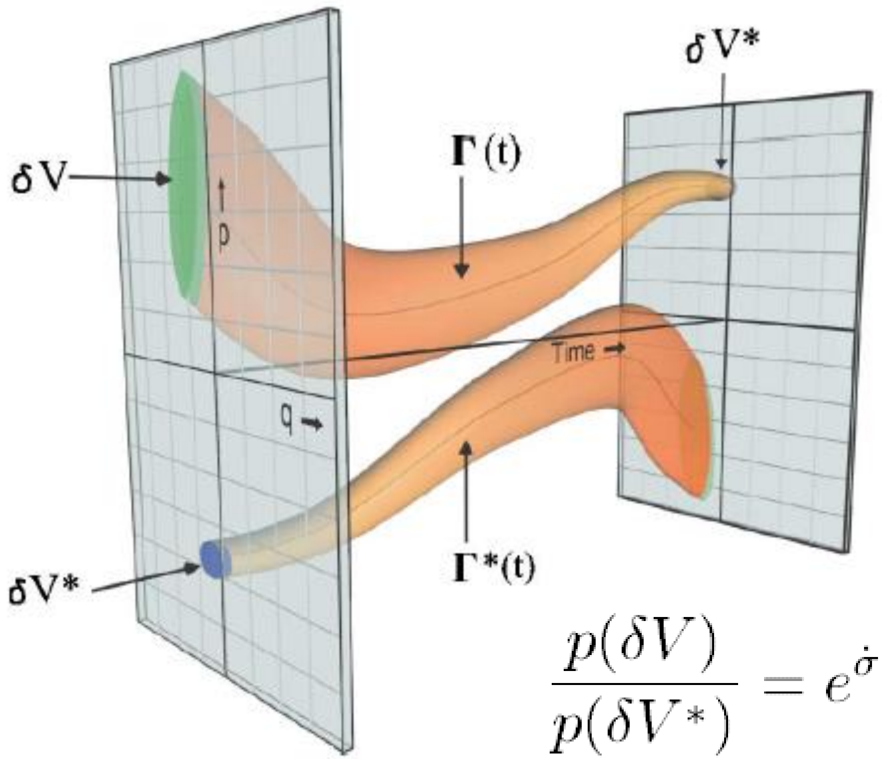
New Theories in Statistical Mechanics

Fluctuation Theorem

- Second law of thermodynamics
- Fluctuation dissipation theorem

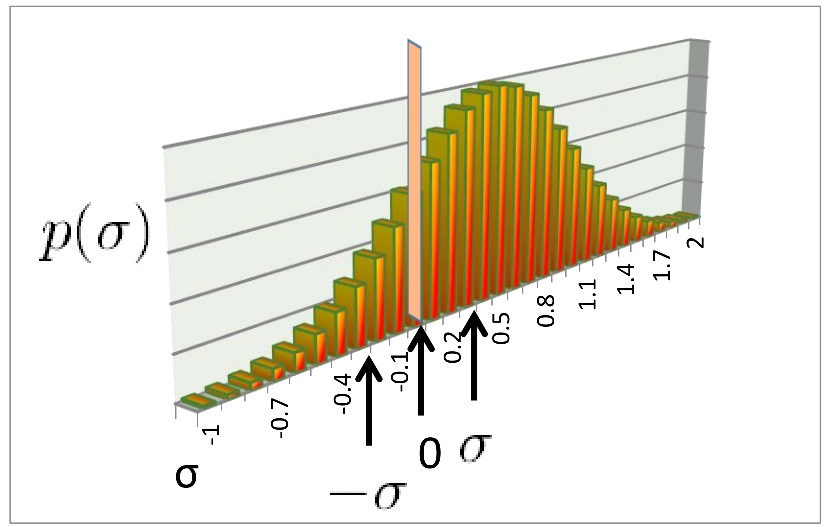
$$\frac{p(\sigma)}{p(-\sigma)} = e^{\sigma}$$

Evans, 1993
Gallavotti, Cohen



$$\frac{p(\delta V)}{p(\delta V^*)} = e^{\dot{\sigma} t}$$

Entropy production: $\sigma = \beta(W - \Delta F)$



Initial conditions producing positive entropy production is much more frequent than the negative entropy production

Confirmed for driven Brownian particles, electric current, etc.

$$\frac{p(\sigma)}{p(-\sigma)} = e^{\sigma}$$

Fluctuation Theorem → **Jarzynski equality**

W :Work performed to the system

F :Free energy gain of the system

$$\langle e^{-(W-\Delta F)/k_BT} \rangle = 1$$

$$e^{-x} \geq 1 - x$$

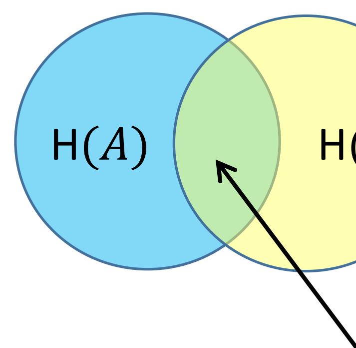
2nd law of thermodynamics: $\langle W \rangle - \Delta F \geq 0$

Generalized Jarzynski equality including information

$$\langle W \rangle \geq \Delta F - kT\langle I \rangle$$

I : mutual information

measurement and control have errors



Correspondingly generalized Jarzynski equality:

$$\Rightarrow \langle e^{(\Delta F - W)/k_BT} \rangle = \gamma$$

Sagawa & Ueda, PRL (2010)

Maxwell's demon

- Violation of the second law of thermodynamics

(1871)

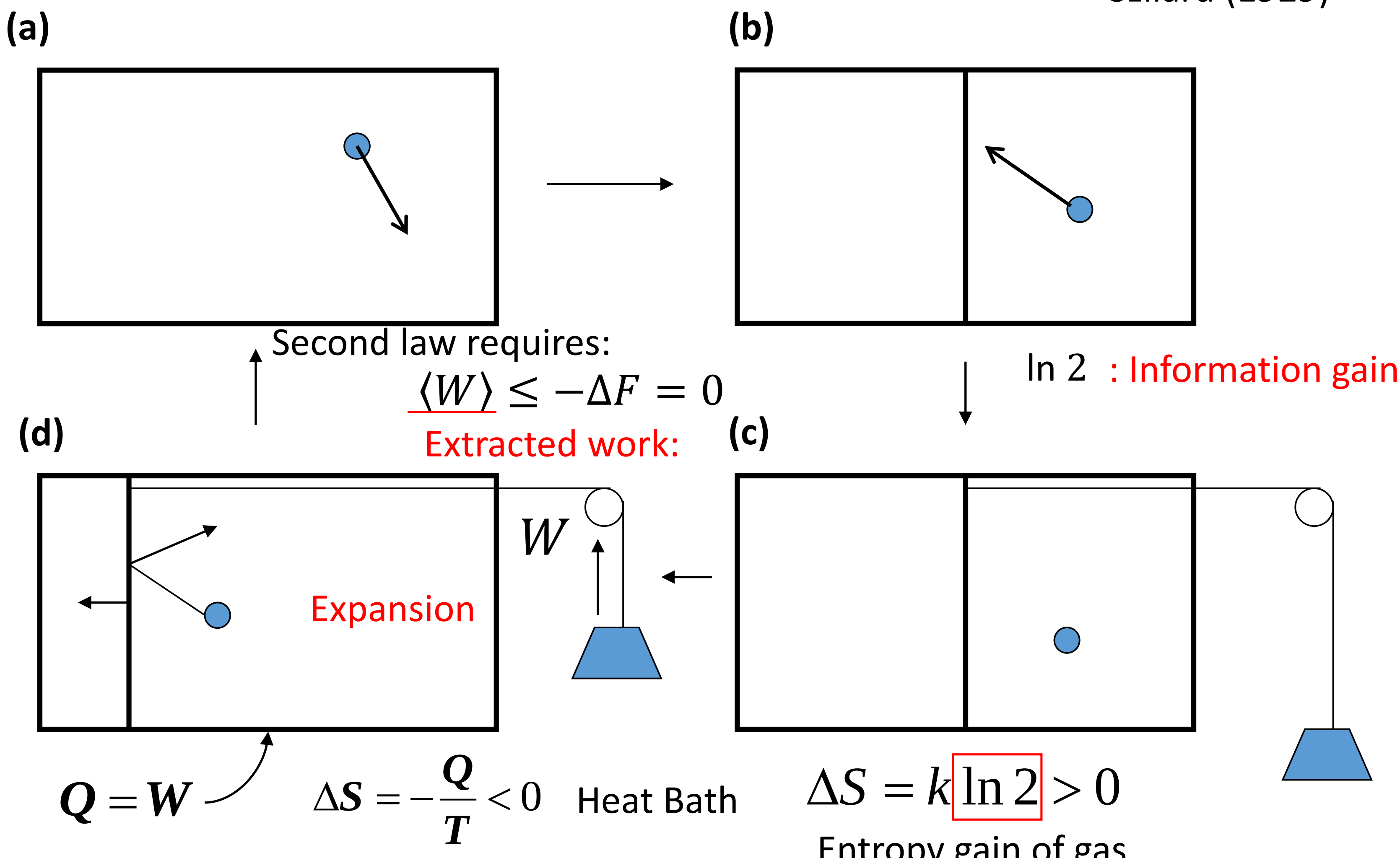
James Clerk Maxwell (1831-1879)

Opening & closing door do not perform work to atoms.
⇒ 2nd law really violate？
⇒ controversial state lasted more than 150 years.

Maxwell's Demon and Szilard Engine

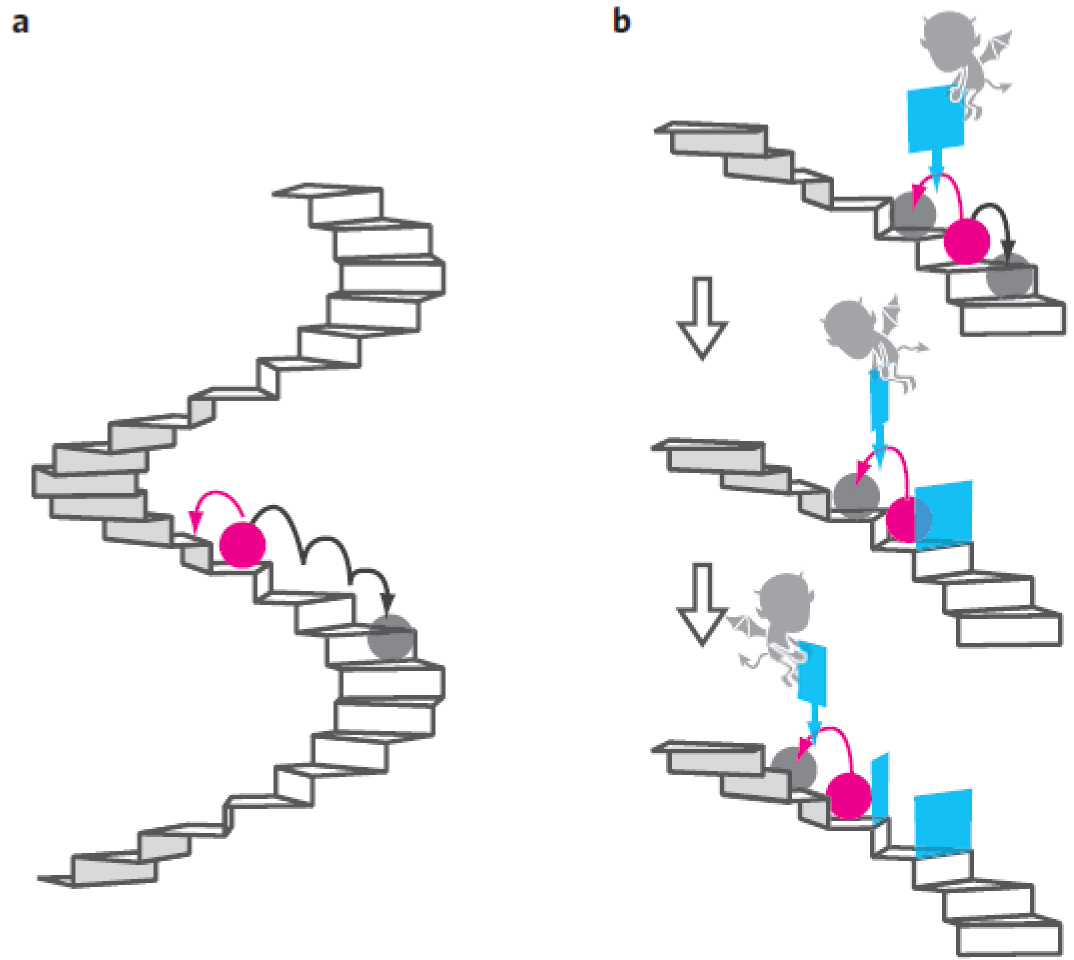
The simplest and analyzable Maxwell's demon

Szilard (1929)



However we gain, $W = k_B T \ln 2$: Information gain → decrease of entropy

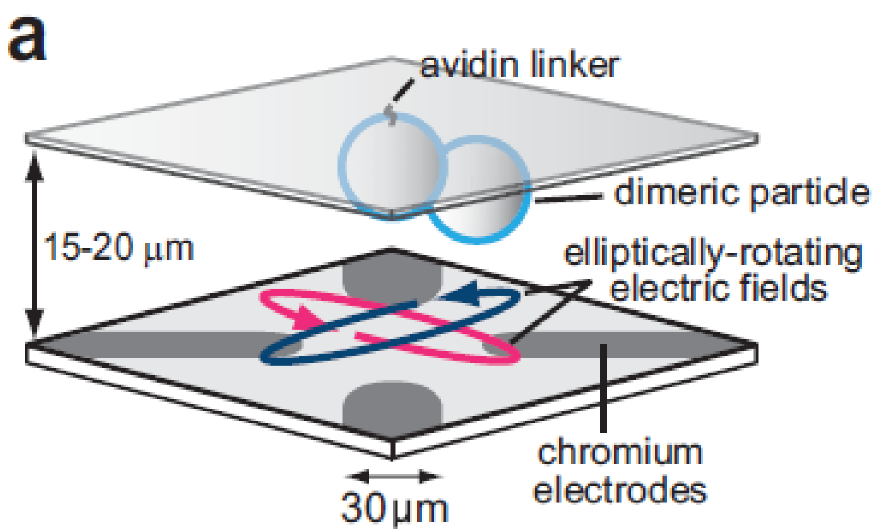
Schematic illustration of the experiment



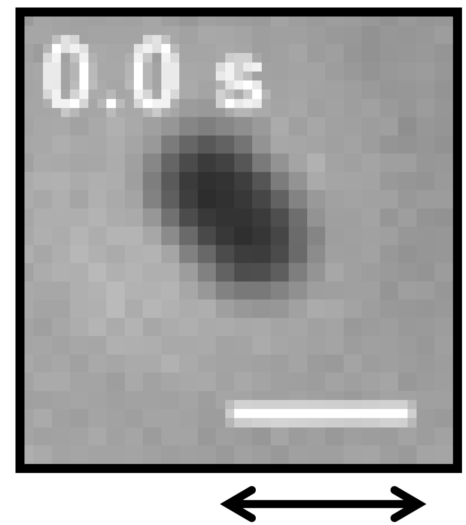
Toyabe, Sagawa, Ueda, Muneyuki, Sano, Nature Physics, 6**,** 988**,** (2010))

Experimental Setup

- Dimeric polystyrene particle (300nm) is linked on the substrate with a biotin.
- Particles exhibit a rotational Brownian motion.

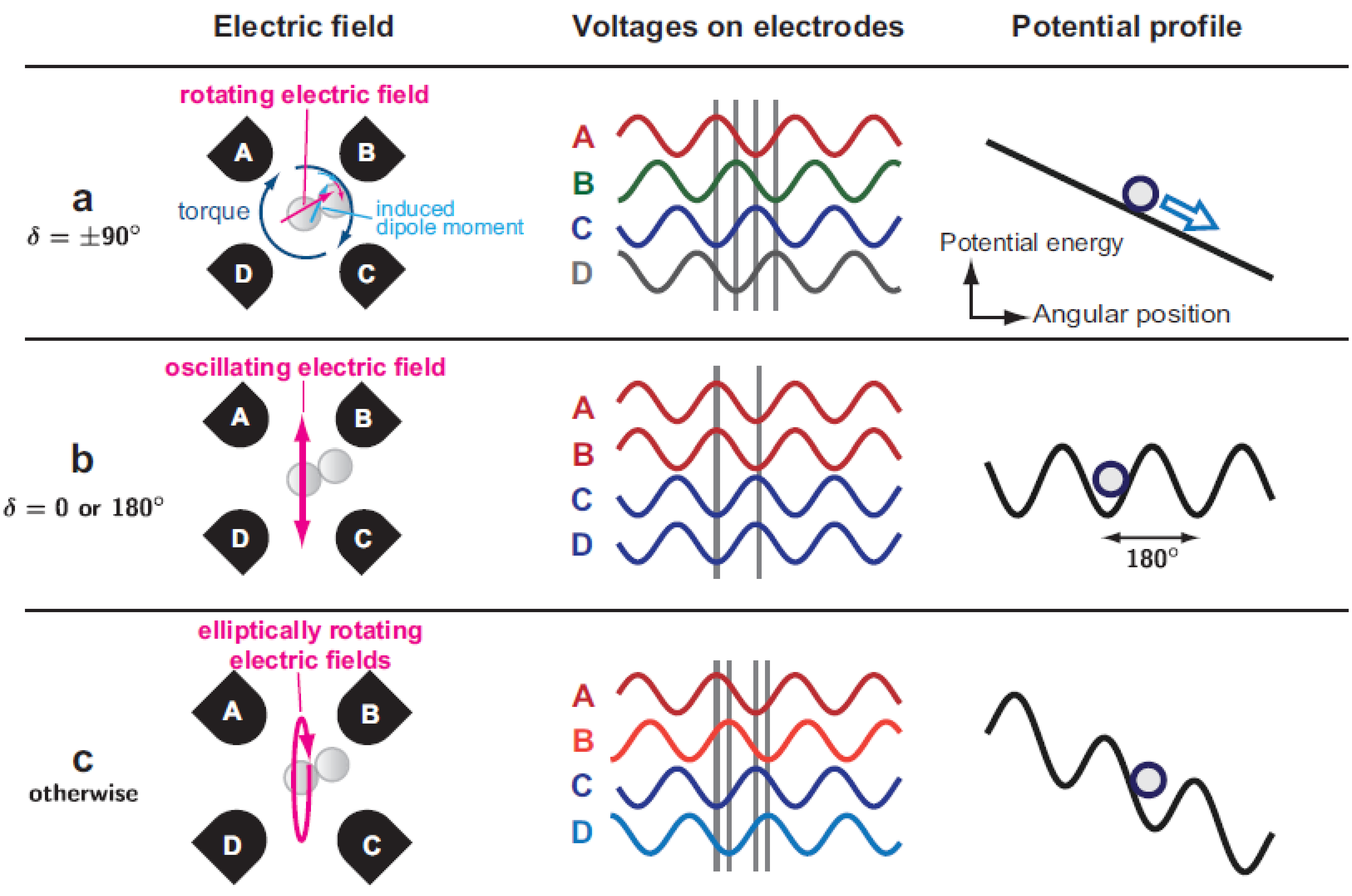


- Quadrant electrodes are patterned on the substrate

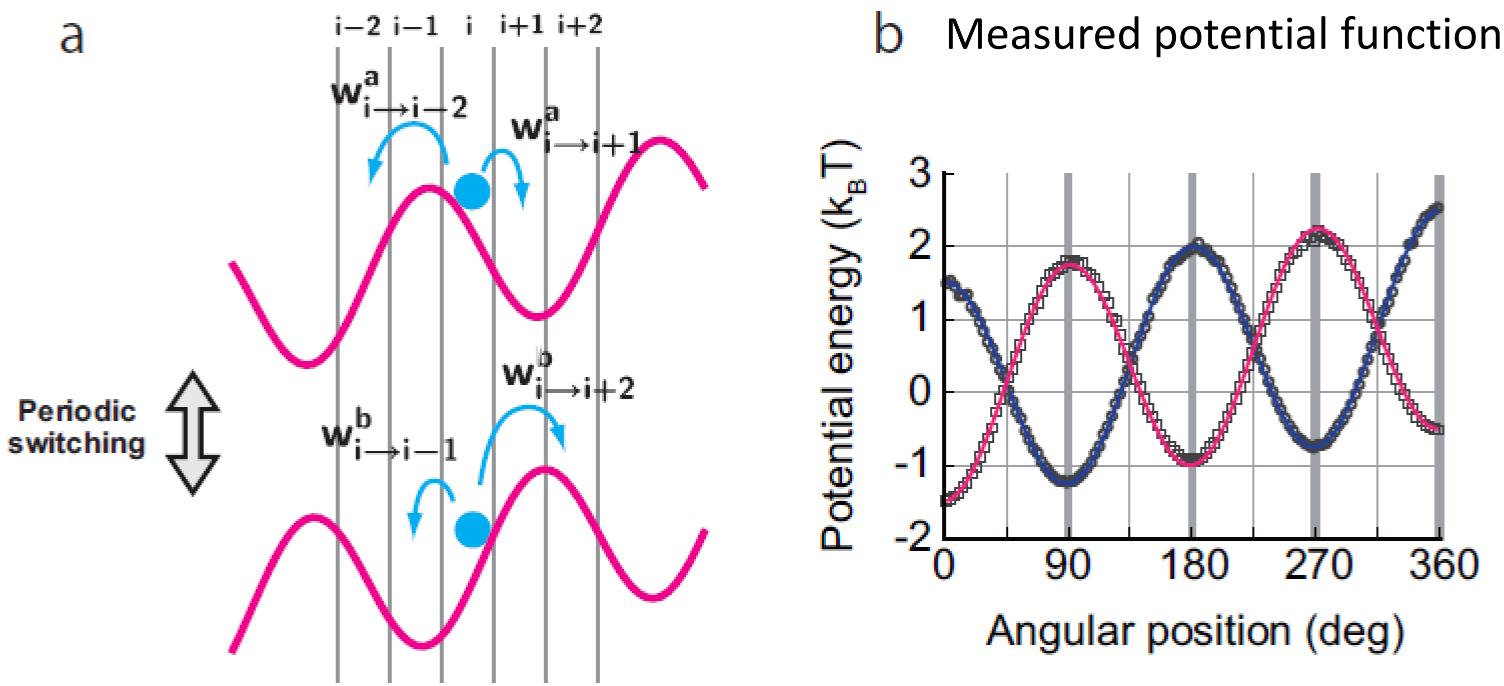


1 µm (1/1000 mm)

How to produce a spiral-stair-like potential



Estimating a potential function from the data

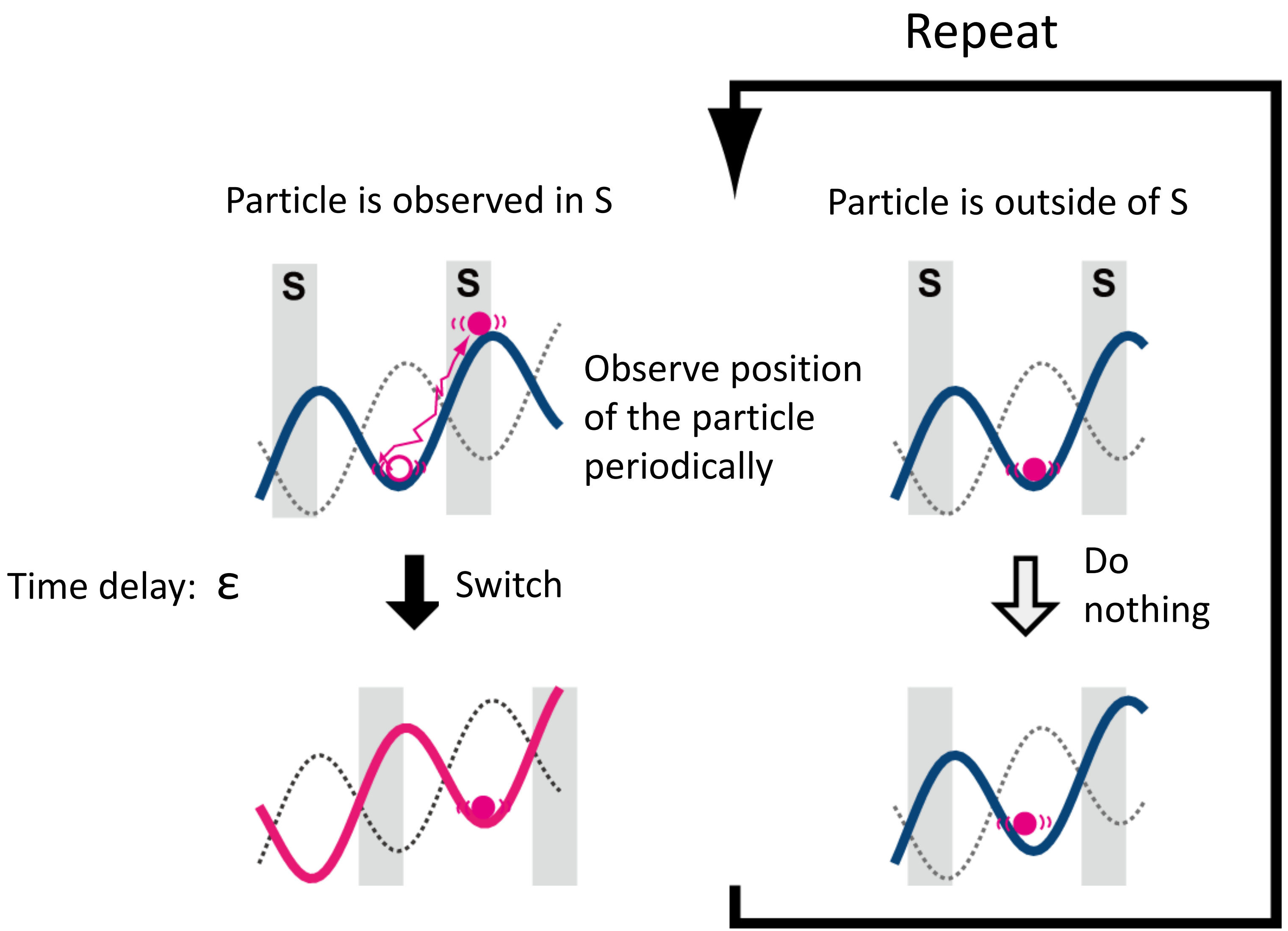


$$\varepsilon^2(\{U_i\}) \equiv \sum_{i<j} \sqrt{n_{i \to j} n_{j \to i}} \left[\Delta U_{i \to j} - \Delta U'_{i \to j}\right]^2,$$

where $\Delta U'_{i \to j} \equiv k_B T \left[\ln w_{j \to i} - \ln w_{i \to j}\right]$

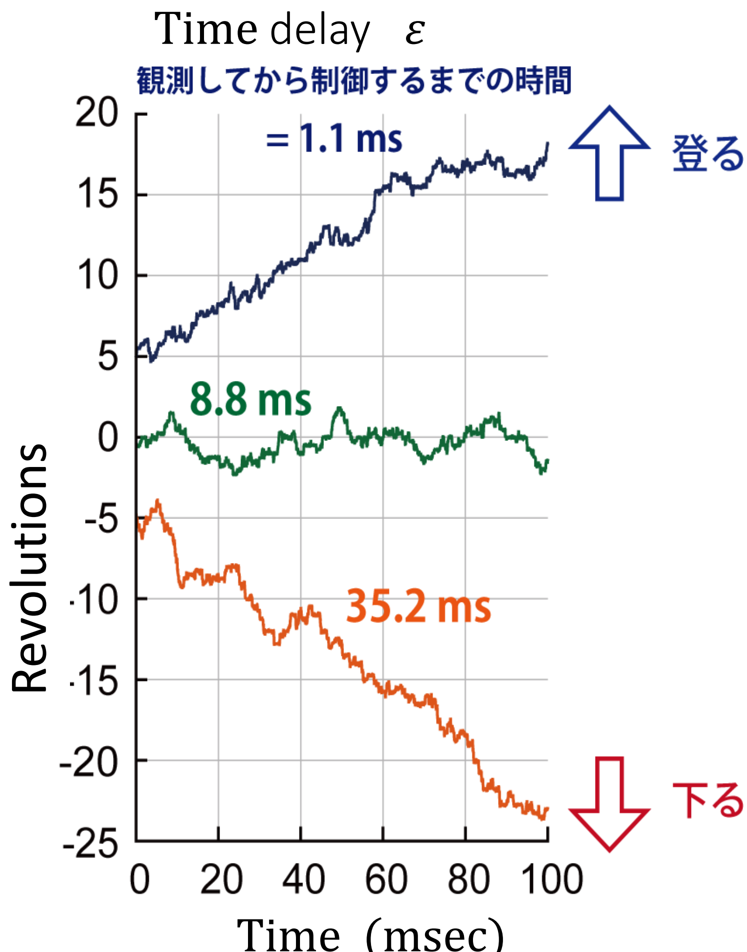
Minimize ε^2

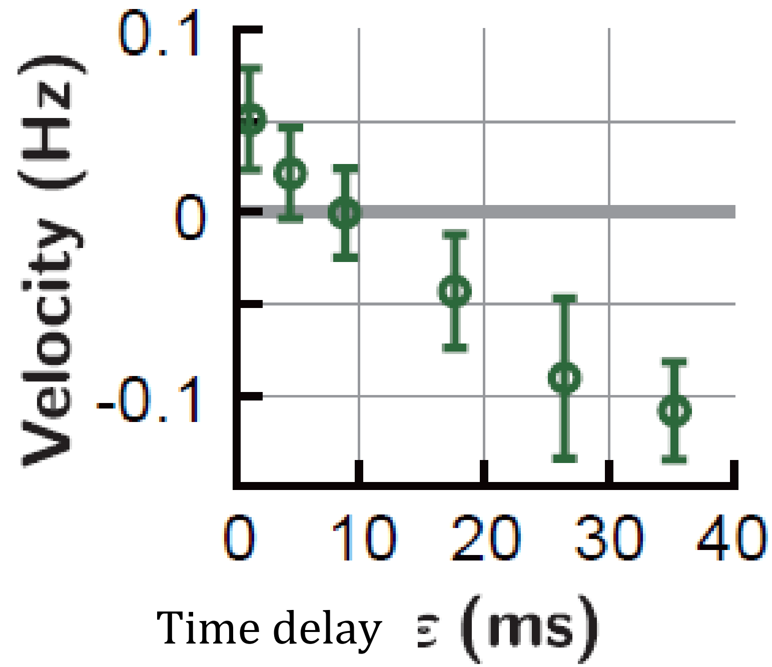
Feedback control based on information contents

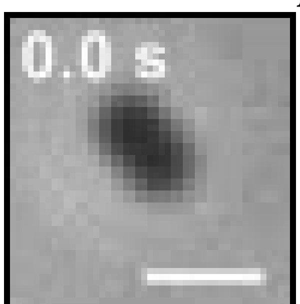


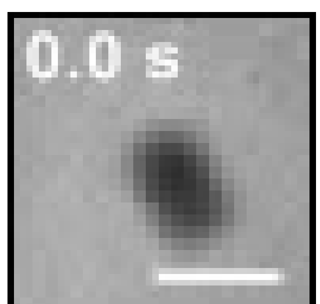
Feedback by Information

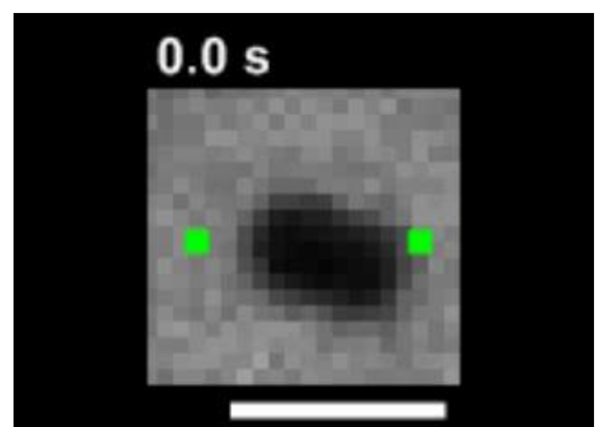
Trajectories under feedback control





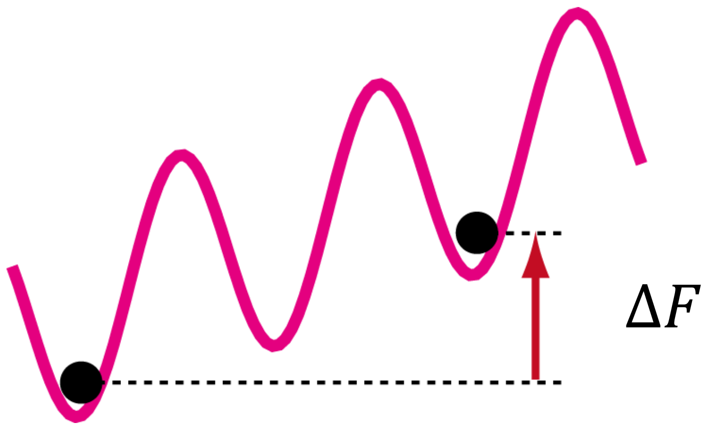






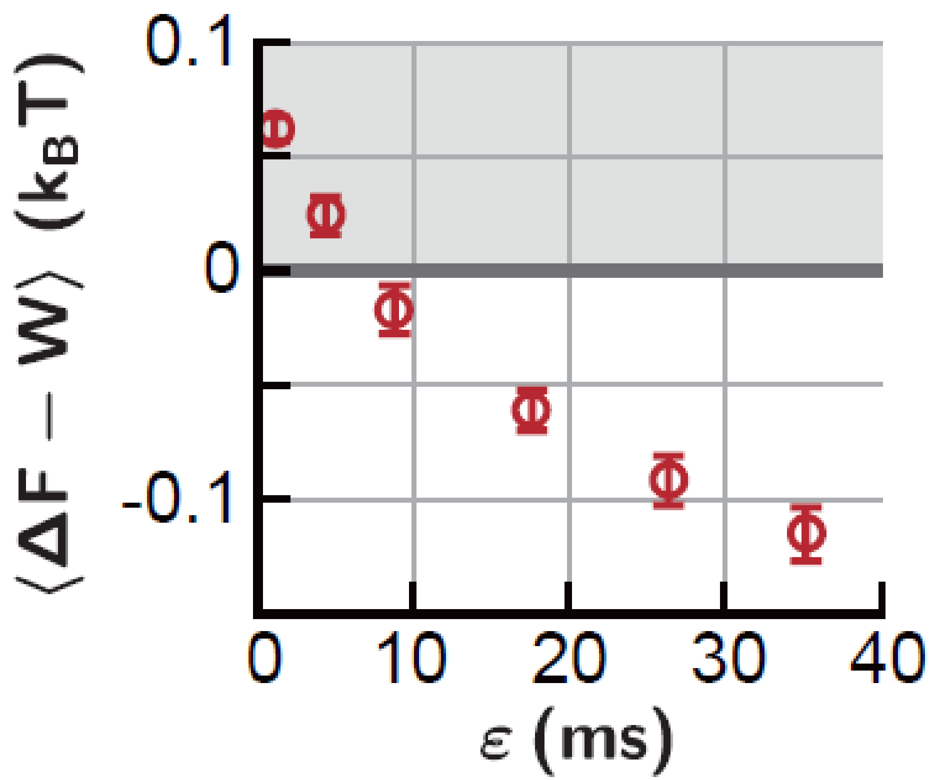
Calculation of Free Energy

- Free energy gain

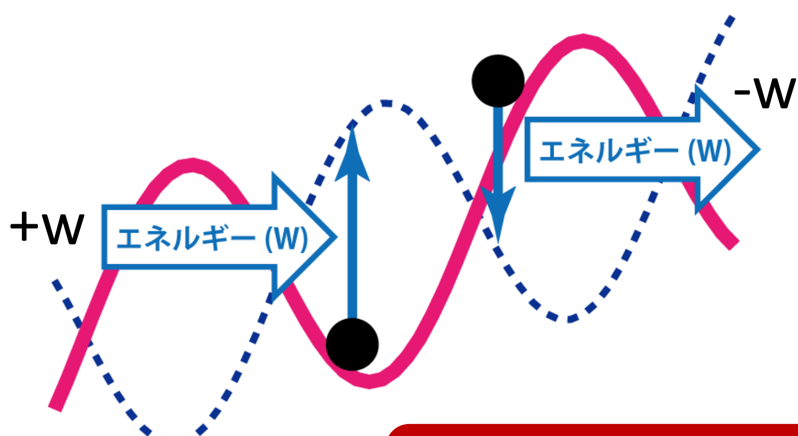


W :Work performed to the system

F :Free energy gain of the system



- Work done to the particle

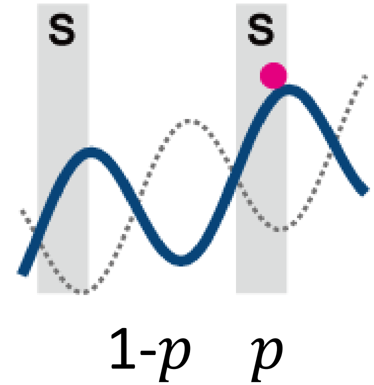


Calculate（ΔF-W）

Efficiency of Information-Energy Conversion

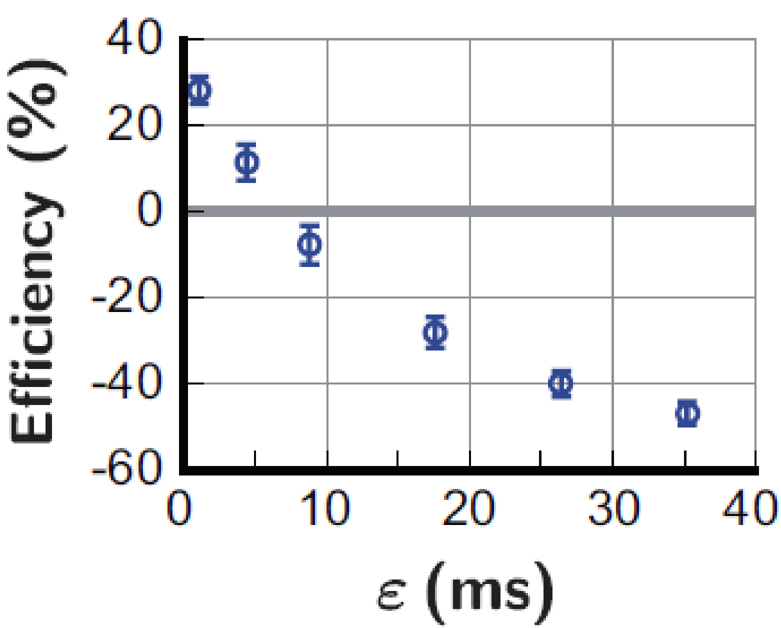
Information gained by the observation:

$$I = -p \ln p \; - (1-p)\ln((1-p)$$



Efficiency of Information-Energy Conversion :

$$\vartheta = \frac{\Delta F - W}{k_B T I}$$



28% Efficiency

Experimental test of generalized Jarzynski equality

- Entropy Production

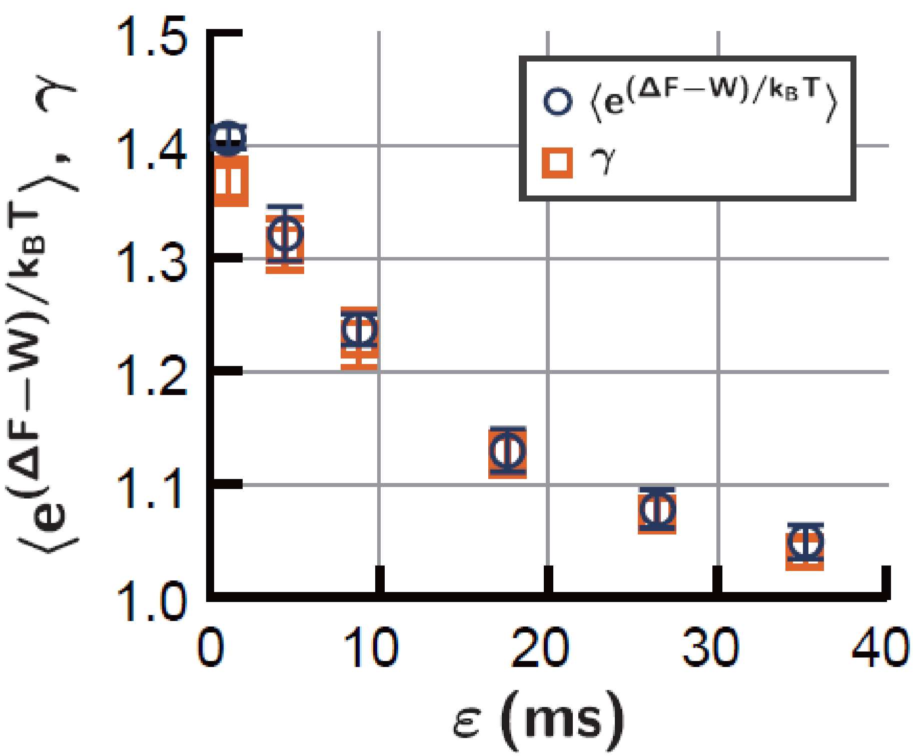
$$\left\langle e^{\beta(\Delta F - W)} \right\rangle = \gamma$$

generalized Jarzynski equality

Sagawa, Ueda, PRL (2010)

Feedback Efficacy

- **A fundamental principle to relate energy and information feedback**

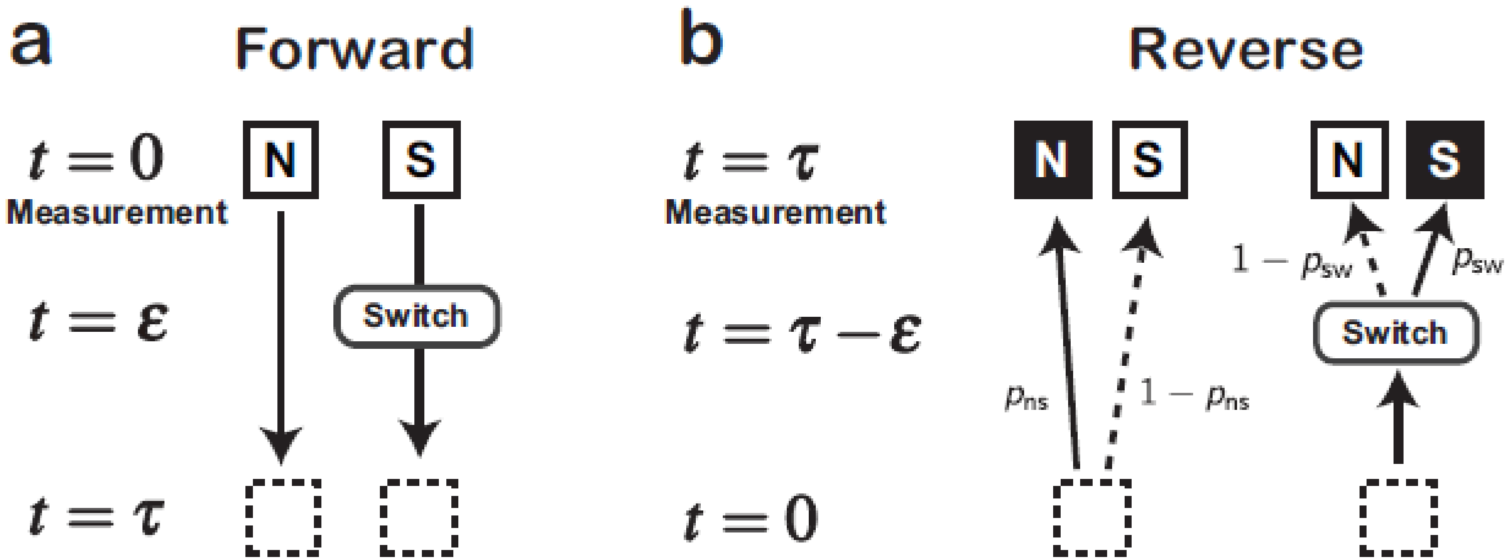


Agrees within measurement accuracy

Toyabe, Sagawa, Ueda, Muneyuki, Sano,

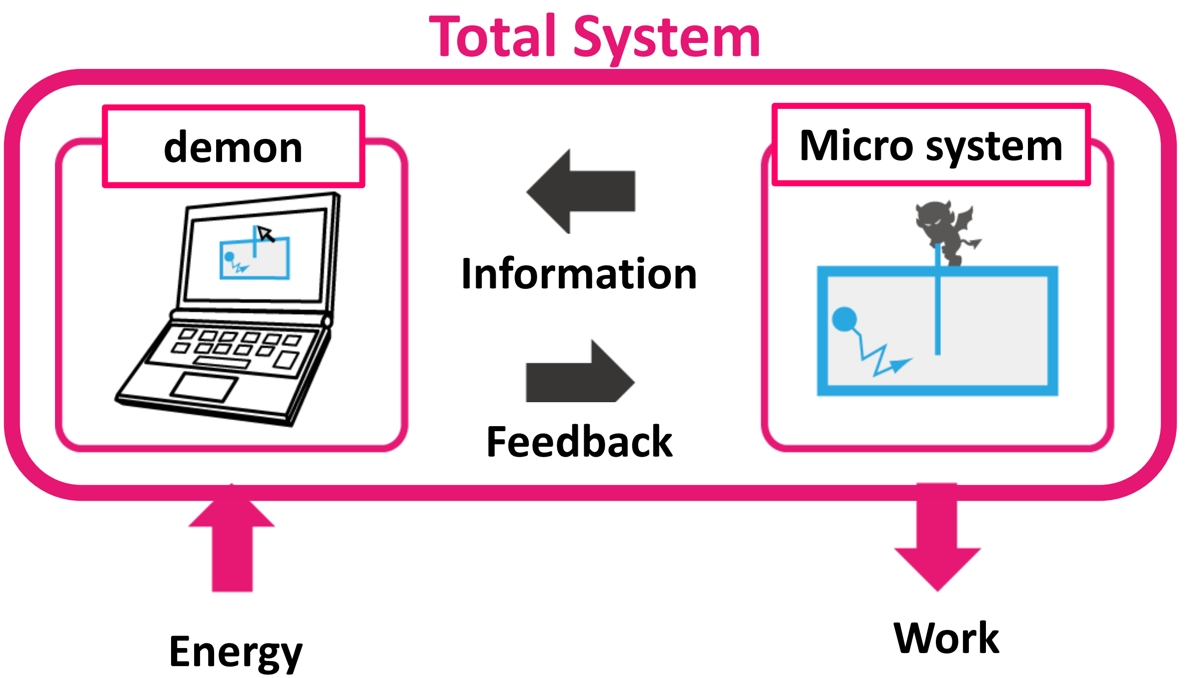
Nature Physics, 6**,** 988**,** (2010))

How to measure the feedback efficacy



$\gamma = p_{sw} + p_{ns} \leq\geq 1$

Consistency with 2nd Law



- 2nd law holds for the total system.
- Information-energy conversion is realized when we look at the small system.

Self-propelled dynamics of Janus particle

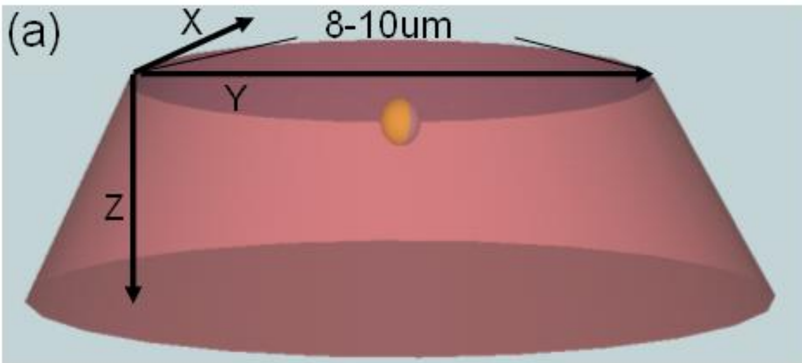
- By a local temperature gradient:

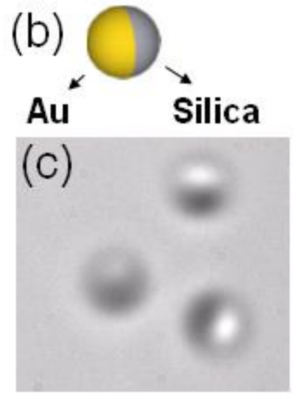
  Self-Thermophoresis

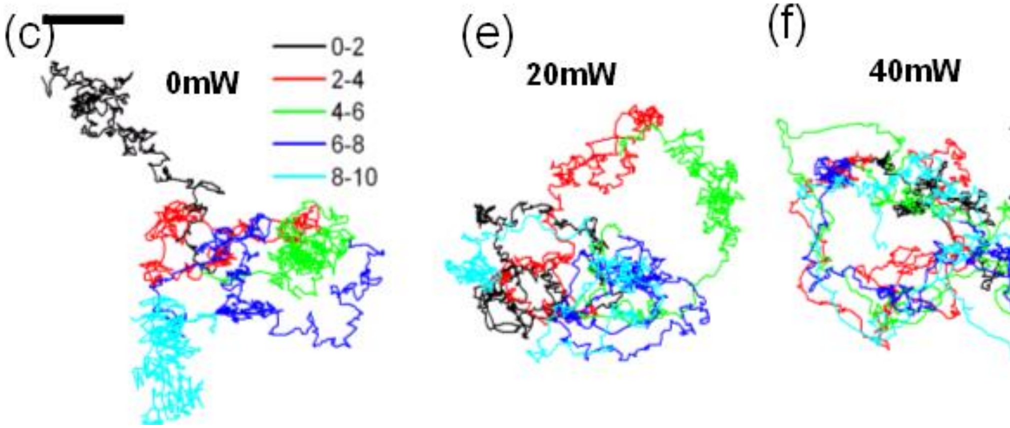
- By electric field:

  Induced Charge Electro-Osmosis (ICEO)

Self-propulsion of Janus particle I : Temperature field



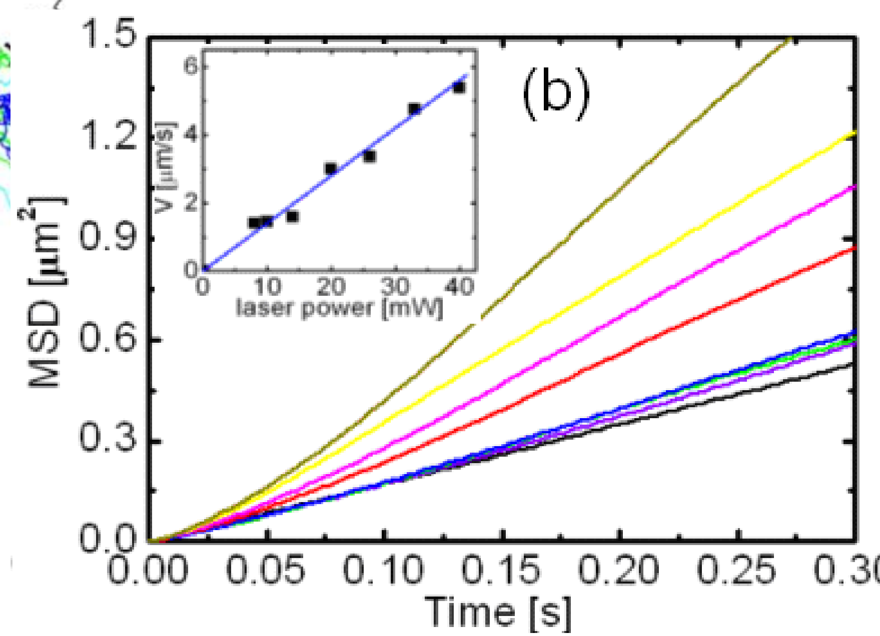




$$\left\langle \Delta r^2(t) \right\rangle = 2\tau V^2[t - \tau(1 - e^{-t/\tau})]$$

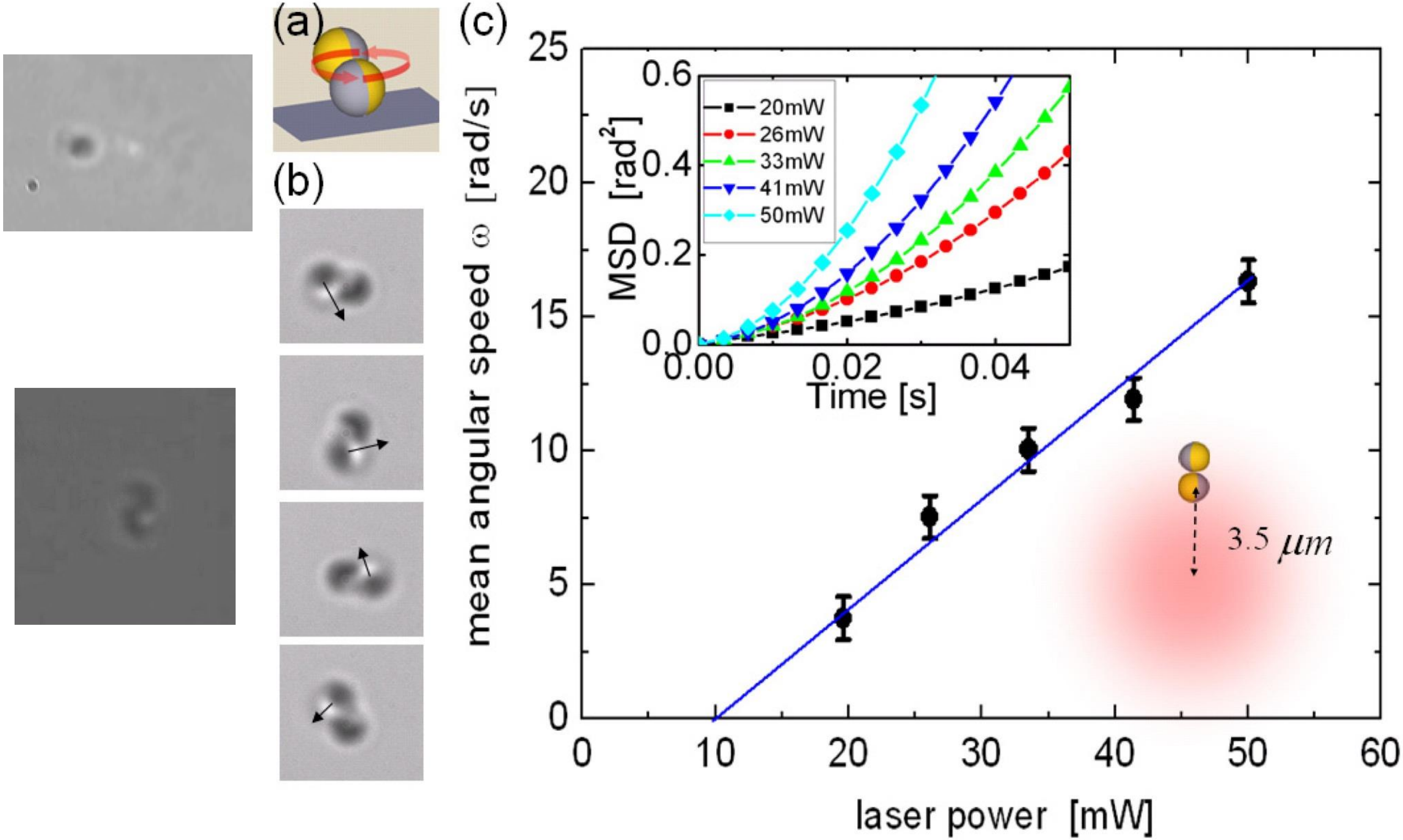
$$\left\langle \Delta r^2(t) \right\rangle \sim V^2 t^2, \qquad t << \tau$$

$$\left\langle \Delta r^2(t) \right\rangle \sim \tau V^2 t, \qquad t >> \tau$$



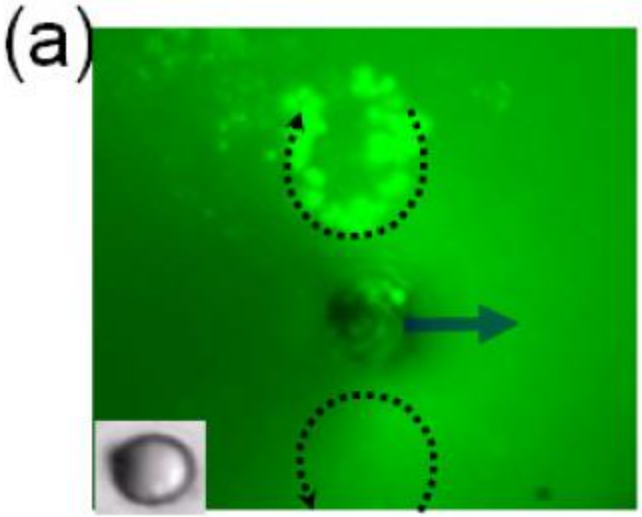
HR Jiang, Yoshinaga, Sano, PRL, 105, 268302 (2010)

Rotation of Chiral Doublet: Thermophoresis

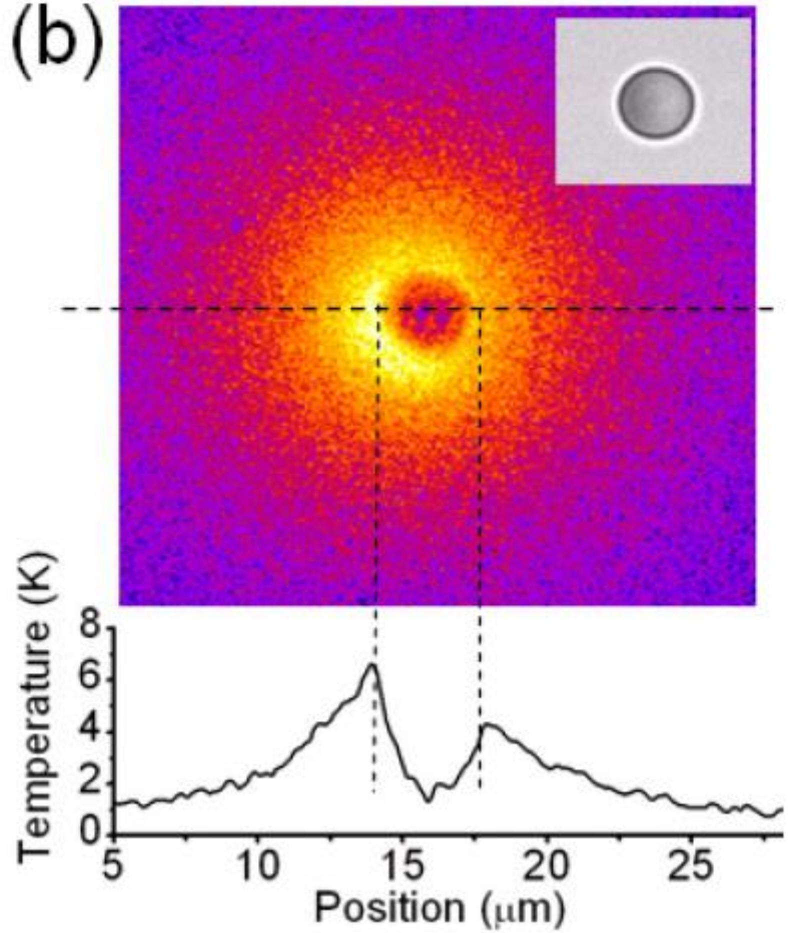


HR Jiang, Yoshinaga, Sano, PRL (2010)

Temperature distribution around a Janus Particle



Induced flow visualized by tracer particles around the fixed Janus particle



Temperature distribution :

$$T(R) = T_\infty + \sum_{n=1}^{\infty} \frac{q_n R}{(n+1)\kappa_o + n\kappa_i} P_n(\cos\theta).$$

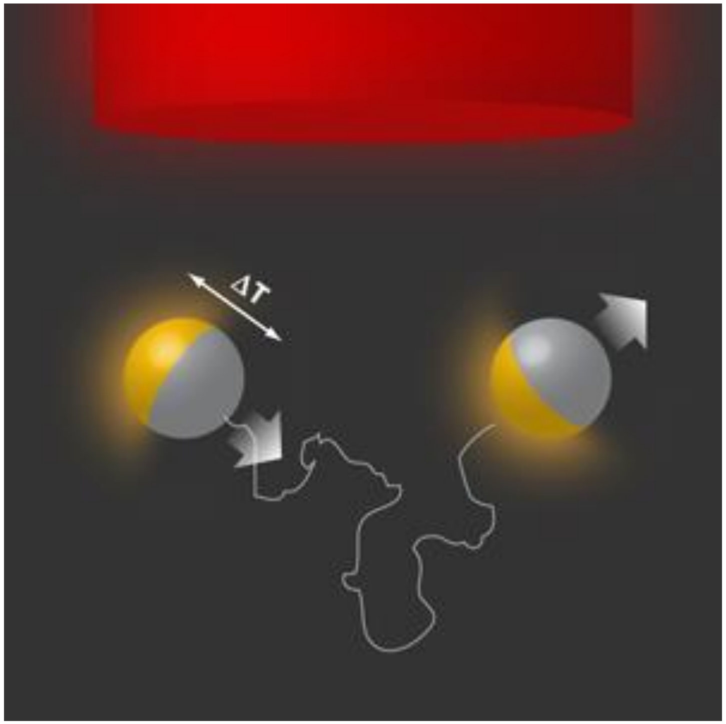
$$q(\theta) = \kappa \mathbf{e}_n \cdot \nabla T$$

$$\Delta T = 3\epsilon I R / 2(2\kappa_o + \kappa_i)$$

Seed of self-thermophoresis of Janus particle is related to the response to the external gradient

$$V = -\frac{1}{4} D (S_T^0 + S_T^G) \frac{\epsilon I}{2\kappa_o + \kappa_i}.$$

Self-thermophoresis



Viewpoint in Physics, PRL (2010)

Light is absorbed on the metal side:

$$-\kappa_o \mathbf{n}\cdot\nabla T_o + \kappa_i \mathbf{n}\cdot\nabla T_i = q(\theta).$$

$$T(R) = T_0 + \sum_{n=0}^{\infty} \frac{q_n R}{(n+1)\kappa_o + n\kappa_i} P_n(\cos\theta)$$

Theoretical calculation by N. Yoshinaga

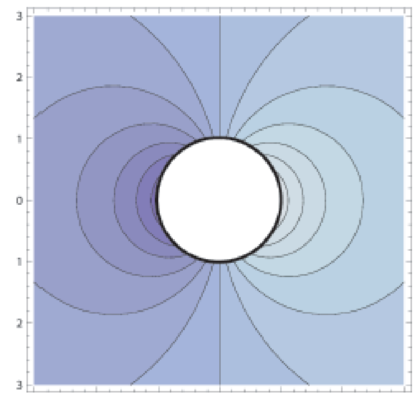
Effective slip velocity: $\boldsymbol{v}_s = v_s \boldsymbol{e}_\theta = \mu |\nabla T|_s \boldsymbol{e}_\theta$

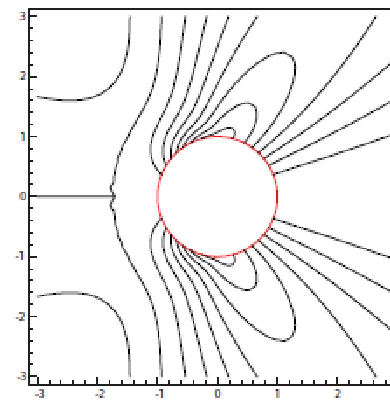
$$\mu = -(k_B/\eta)\Gamma\lambda$$

Characteristic length: $\lambda = \Gamma^{-1} \int c_0 y (e^{-\beta U_0} - 1) dy$

U_0 : Interaction potential between the surface and fluid

$$\Gamma = \int c_0 (e^{-\beta U_0} - 1) dy \qquad V = -\frac{1}{2}\int_0^{\pi} v_s \sin^2\theta d\theta.$$





Migration in an uniform external temperature gradient

$$V = -\mu T_1 = -D S_T T_1$$

T_1 : Temperature diff. across the particle

$$\mu = D S_T$$

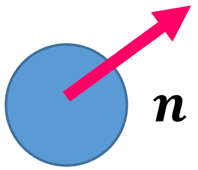
$$V = -\frac{1}{4} D (S_T^0 + S_T^G) \frac{\epsilon I}{2\kappa_o + \kappa_i}.$$

Migration speed is determined by the average of Soret coeff.

Stochastic Dynamics of Active particle

Langevin equation:

$$\gamma \boldsymbol{v} = -\frac{\partial U}{\partial \boldsymbol{r}} + \underline{\beta \boldsymbol{n}} + \boldsymbol{\xi}(t)$$



Fokker-Planck equation:

$$\frac{\partial P(\mathbf{r}, \mathbf{n})}{\partial t} = -\frac{\partial}{\partial \mathbf{r}} \cdot (\mathbf{v}P) - \mathcal{R} \cdot (\omega P)$$

$$\mathcal{R} = \boldsymbol{n} \times \frac{\partial}{\partial \boldsymbol{n}}$$

$$\mathbf{v} = -D \frac{\partial}{\partial \mathbf{r}} (k_B T \ln P + U) + \alpha \mathbf{n}.$$

$$\omega = \frac{1}{\zeta_r} \mathbf{N} = -\frac{1}{\zeta_r} \mathcal{R}(k_B T \ln P + U),$$

$$\frac{\partial P(\mathbf{r}, \mathbf{n}, t)}{\partial t} = \frac{\partial}{\partial \mathbf{r}} \cdot D \left(\frac{\partial P}{\partial \mathbf{r}} + \frac{P}{k_B T} \frac{\partial U}{\partial \mathbf{r}} \right) - \alpha \frac{\partial}{\partial \mathbf{r}} \cdot (\mathbf{n}P) + D_r \mathcal{R} \cdot \left(\mathcal{R}P + \frac{P}{k_B T} \mathcal{R}U \right).$$

$$D = \frac{k_B T}{6\pi\eta a} \qquad D_r = \frac{k_B T}{8\pi\eta a^3}$$

$$U = \frac{1}{2} k r^2 \qquad \mathcal{R}U = 0.$$

Correlation function of the polarity direction:

$$< \mathbf{n}(t) \cdot \mathbf{n}(0) >= \int d\mathbf{n} d\mathbf{n}' d\mathbf{r} d\mathbf{r}' \left[\mathbf{n} \cdot \mathbf{n}' G(\mathbf{n}, \mathbf{n}', t) P_{\mathrm{eq}}(\mathbf{r}', \mathbf{n}') \right]$$

Equilibrium distributionc

$$P_{\mathrm{eq}}(\mathbf{r}, \mathbf{n}) = \frac{1}{4\pi} \left(\frac{k}{2\pi k_B T} \right)^{-3/2} e^{-\frac{k\mathbf{r}^2}{2k_B T}}.$$

Rotational Diffusion:

$$< \mathbf{n}(t) \cdot \mathbf{n}(0) >= \exp(-2D_r t).$$

$$\frac{\partial}{\partial t} < (\mathbf{r}(t) - \mathbf{r}(0))^2 >= \int d\mathbf{r} d\mathbf{n}' d\mathbf{r} d\mathbf{r}' \left[(\mathbf{r} - \mathbf{r}')^2 \frac{\partial G}{\partial t} P_{\mathrm{eq}} \right].$$

MSD of active particle:

$$< (\mathbf{r}(t) - \mathbf{r}(0))^2 >= \frac{6k_B T}{k} + \frac{2\alpha^2}{\tilde{D}} e^{-2D_r t} - \left(\frac{2\alpha^2}{\tilde{D}} t + \frac{6k_B T}{k} + \frac{2\alpha^2}{\tilde{D}} \right) e^{-Dkt/(k_B T)},$$

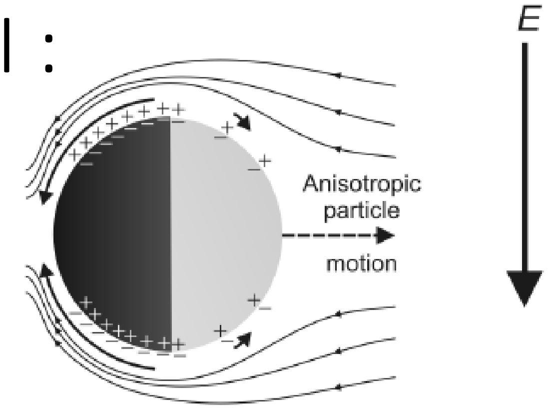
$$\tilde{D} = \frac{-2D_r k_B T + Dk}{k_B T}.$$

Without potential:

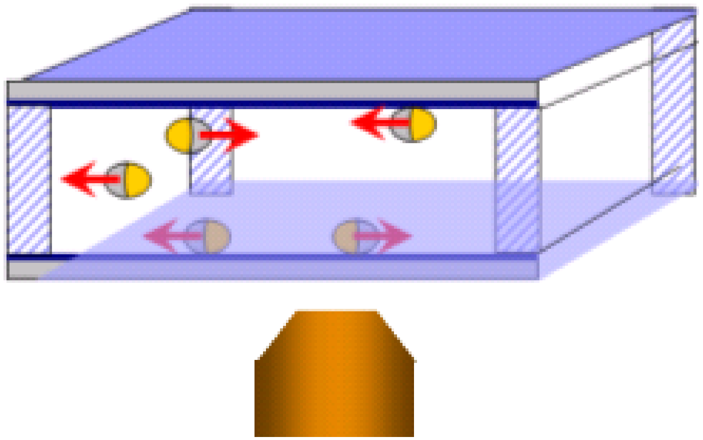
for $k = 0$

$$< (\mathbf{r}(t) - \mathbf{r}(0))^2 >= 6Dt + \frac{\alpha^2}{2D_r^2} \left(e^{-2D_r t} + 2D_r t - 1 \right),$$

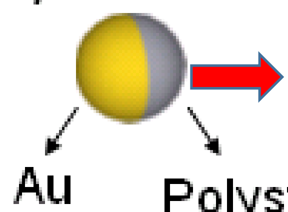
Self-propulsion of Janus particle II :
Electric field



(a)



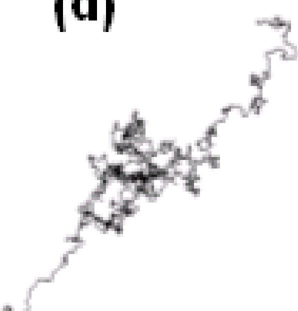
(b)



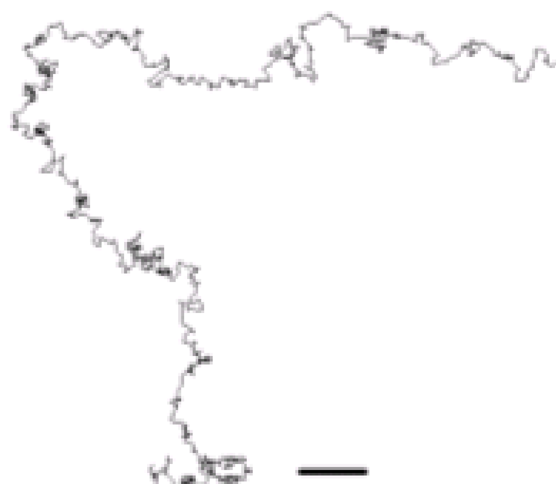
$$V_{ICEO} \sim \frac{\epsilon \varepsilon R E^2}{\gamma}$$

(c)

(d)

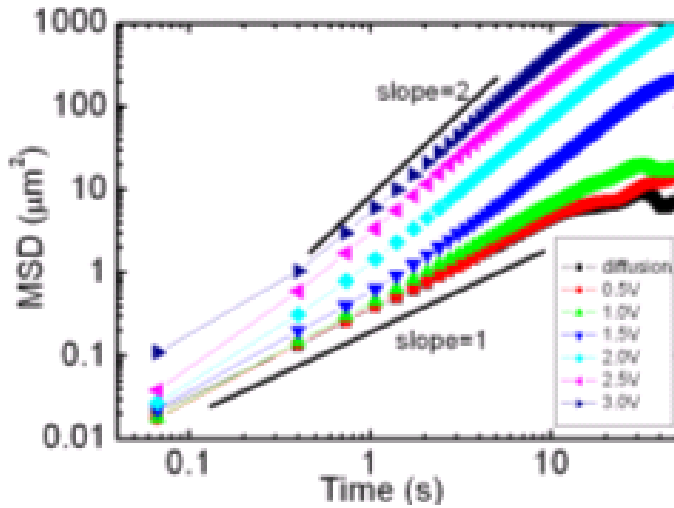


(e)

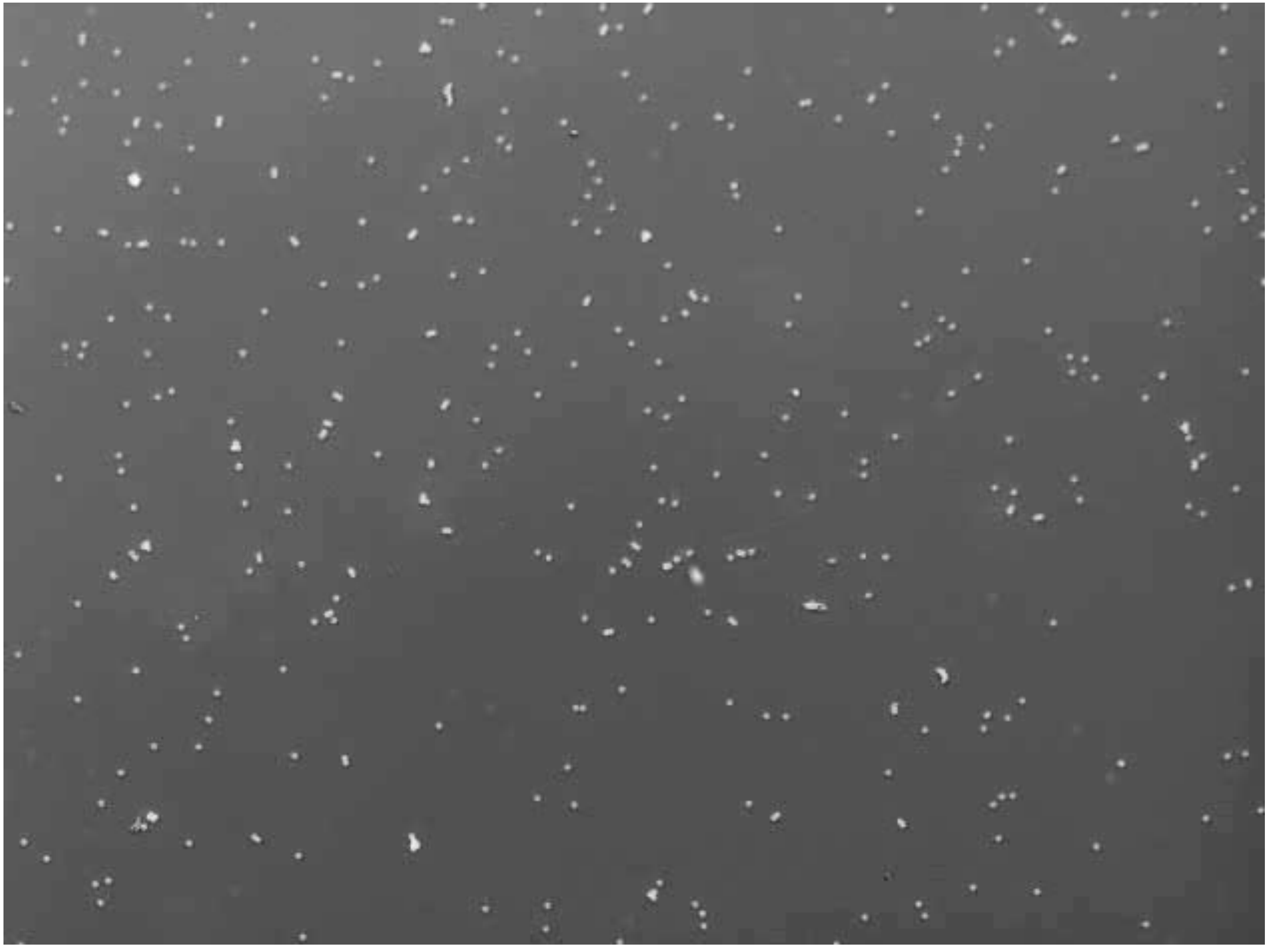


(f)

$$\langle r^2 \rangle = 4Dt + V^2 t^2$$



Motion of Janus Particle: Top View



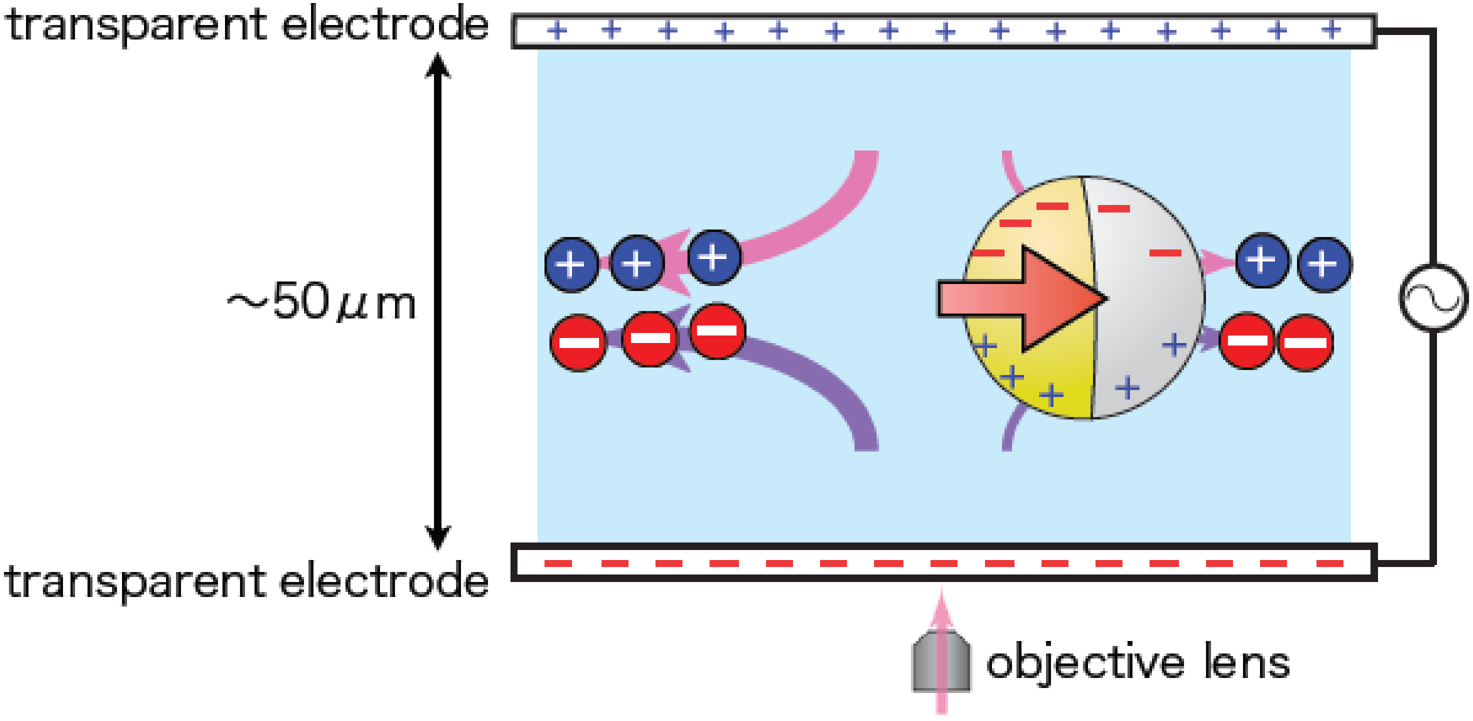
How do they move?

1) Surface charges are induced by the AC electric field $\vec{E}$

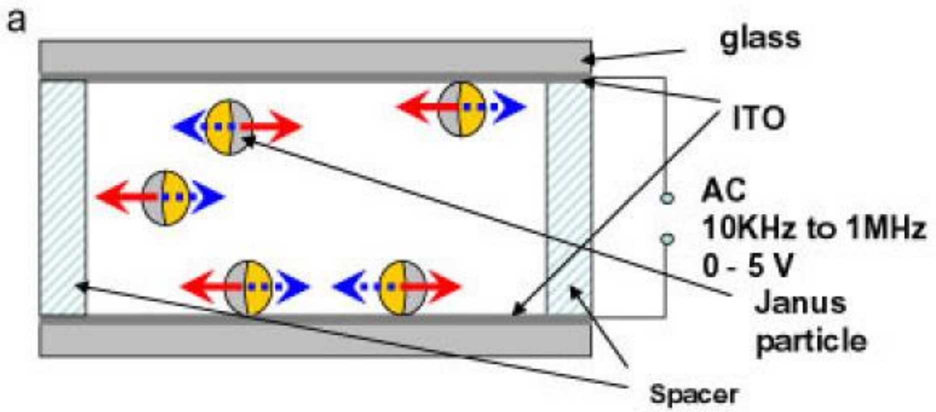
2) Counter-ions gathers around the induced charges.

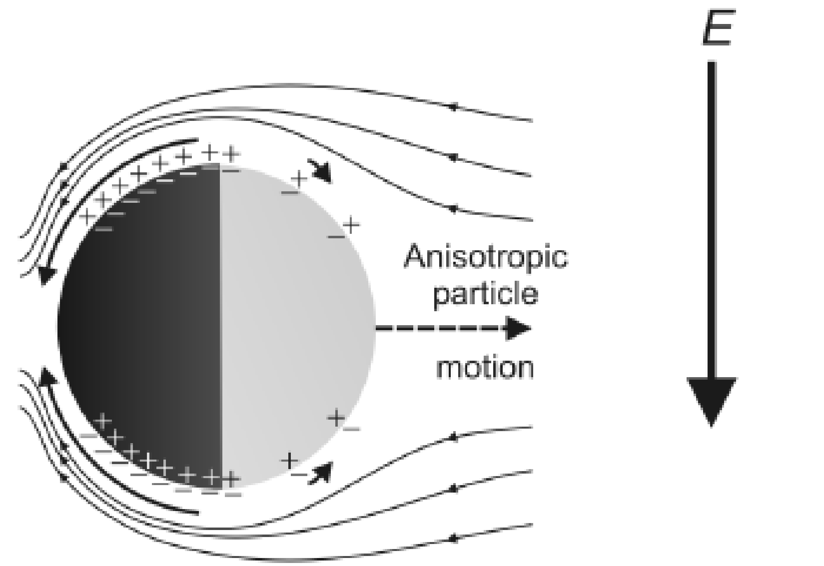
3) Flow of fluid containing counter-ions is induced by $\vec{E}$

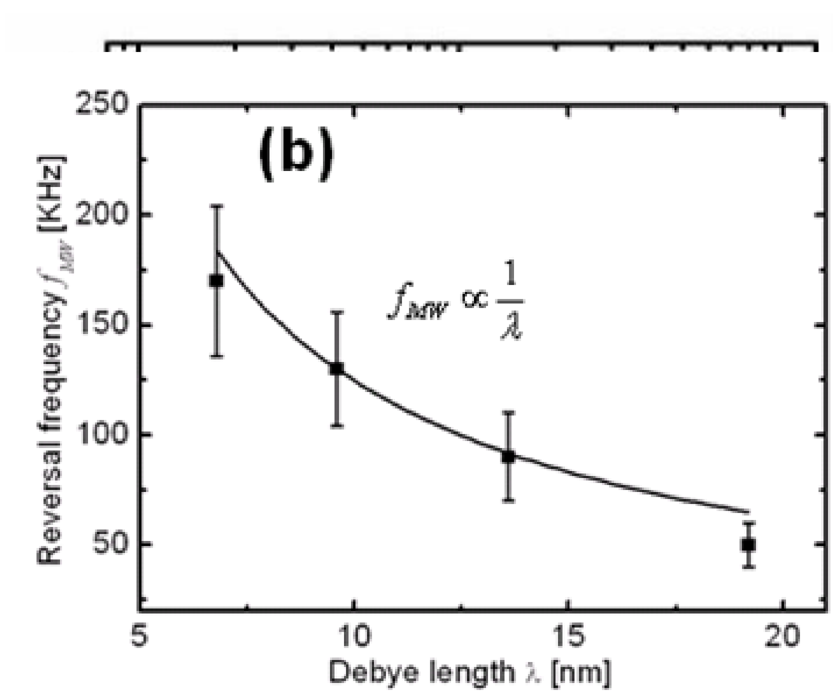
4) Janus particle moves in the opposite direction of the net flow

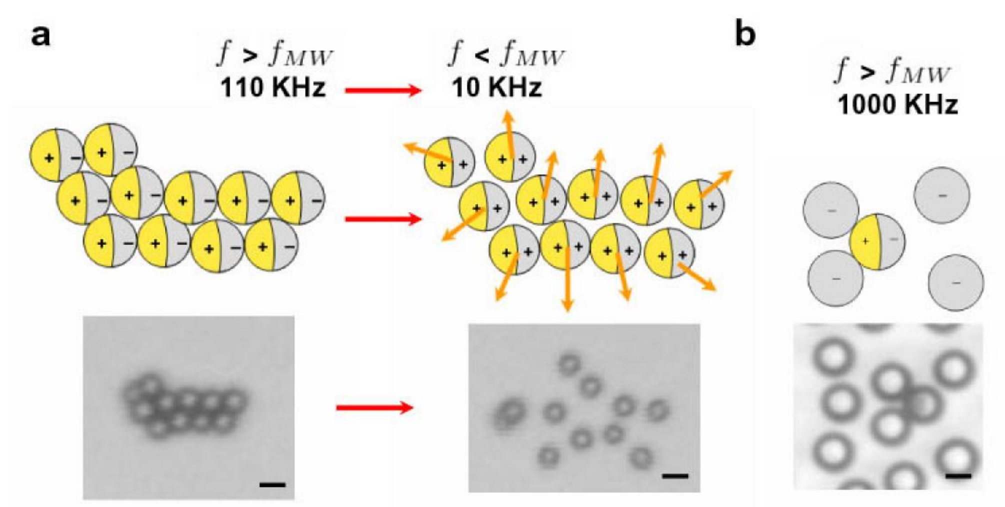


Self-propulsion of Janus particle II :
Electric field

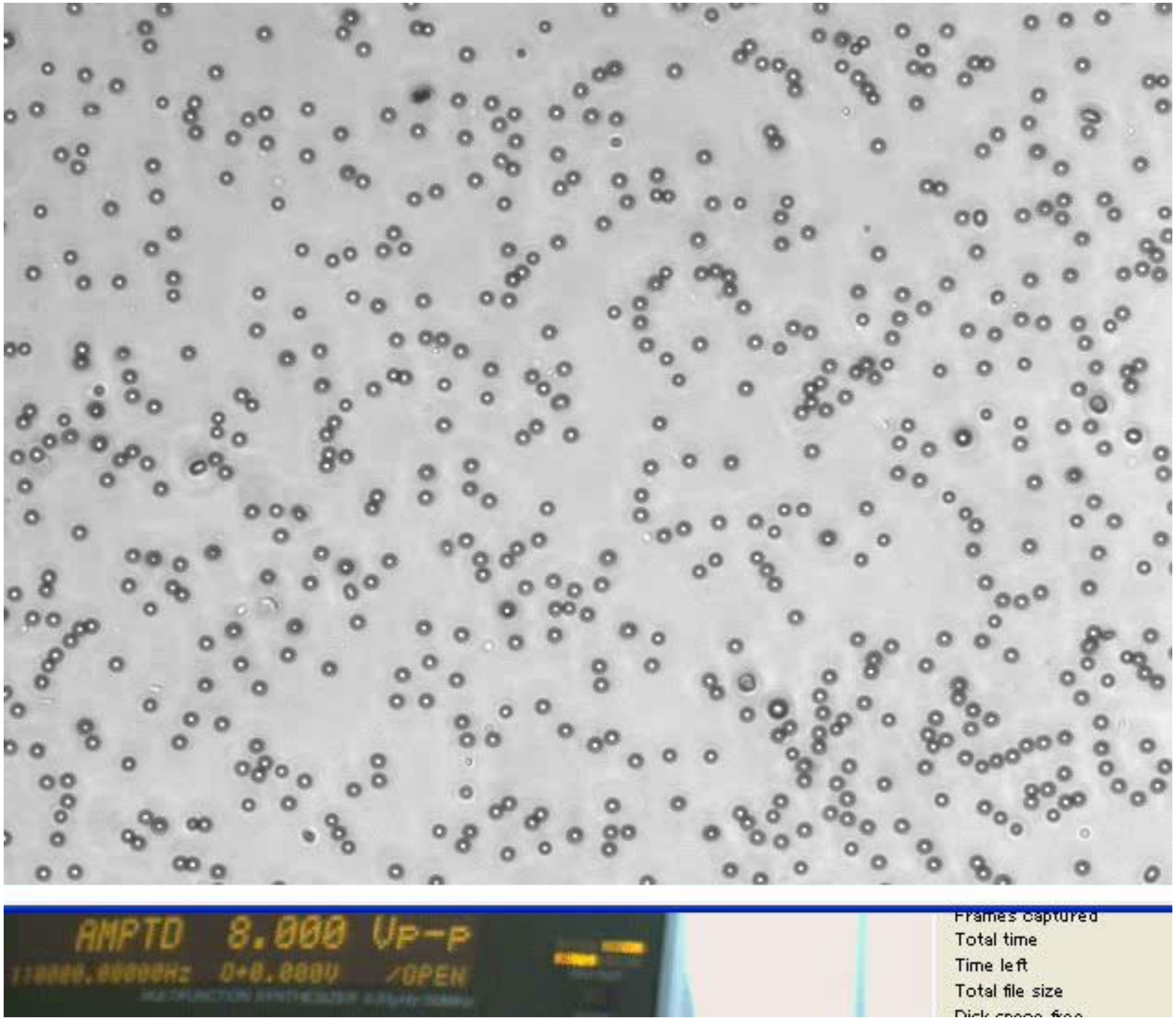


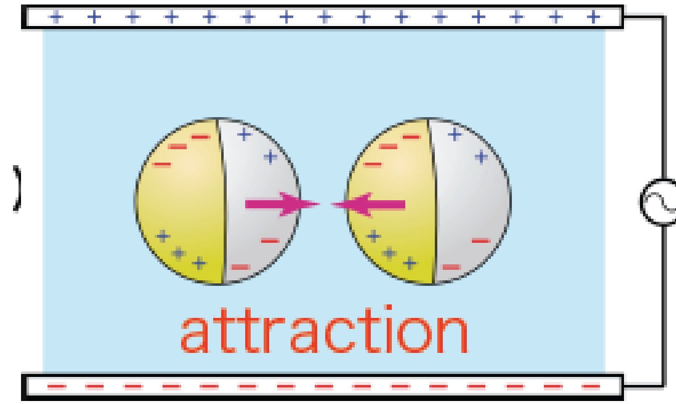


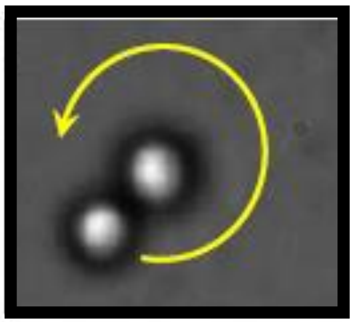




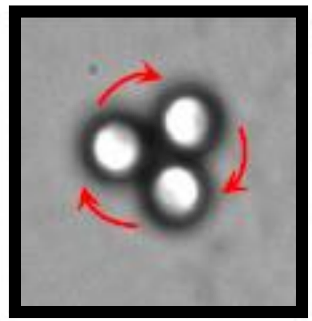
Formation of Chains at frequency higher than fc



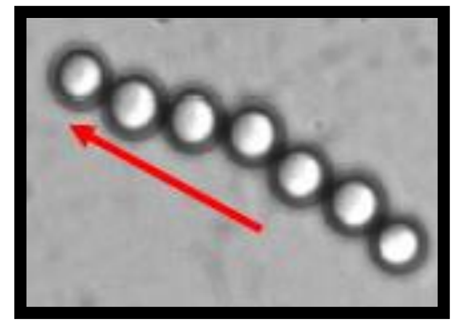




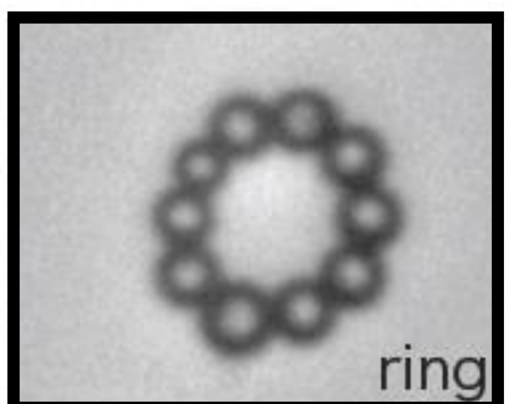
Doublet

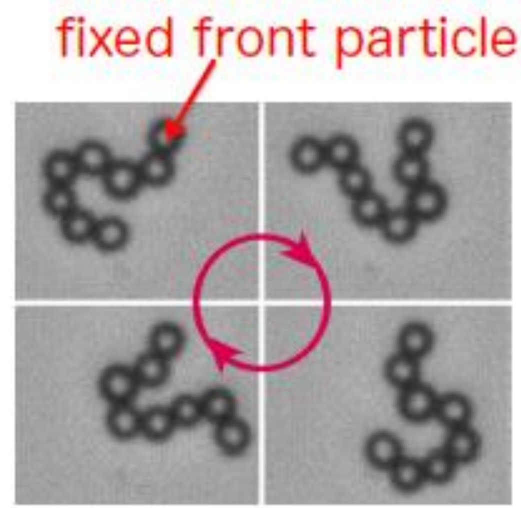


Triplet



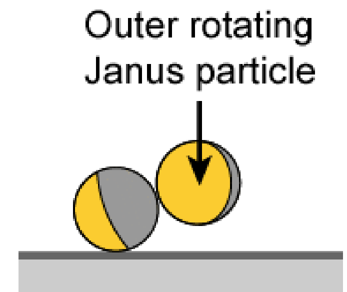
Linear Chain





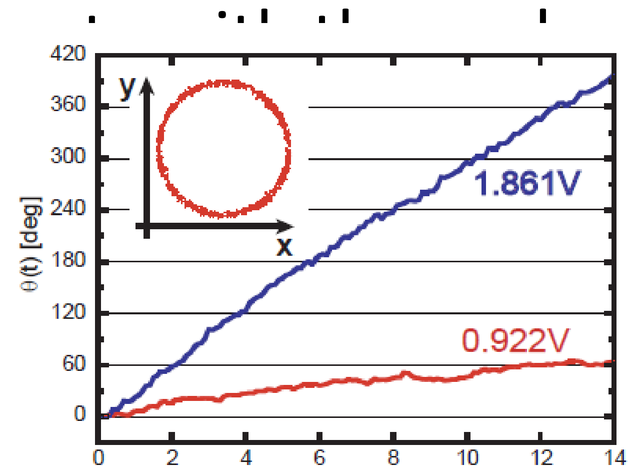
Oscillation, Wave

Fluctuation in Janus Chiral Doublet



Role of Thermal fluctuation

- How does the driving force com... fluctuations?

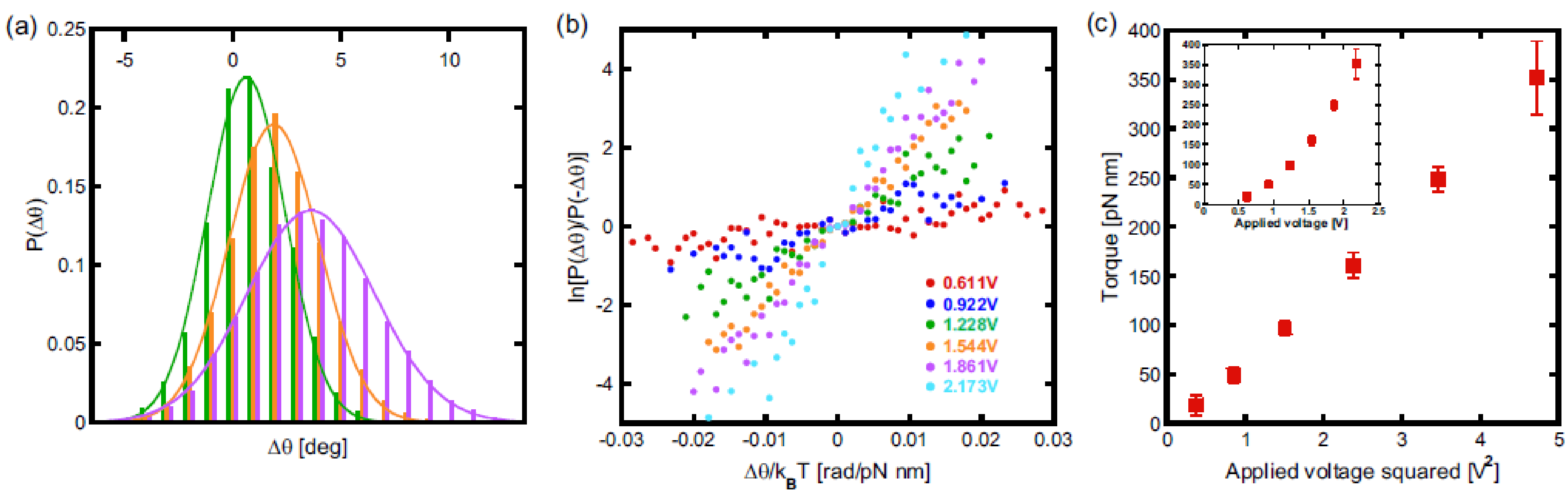


- Use of Fluctuation Theorem

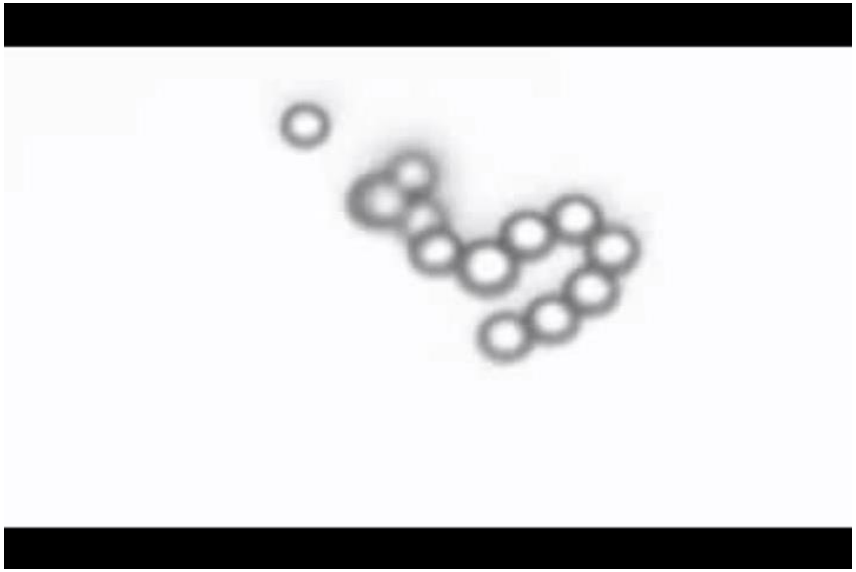
$$\ln\left[P(\Delta\theta)/P(-\Delta\theta)\right] = \tau\,\Delta\theta/k_B T$$

Precise determination of torque is possible

R. Suzuki, HR Jiang, M. Sano, Archive

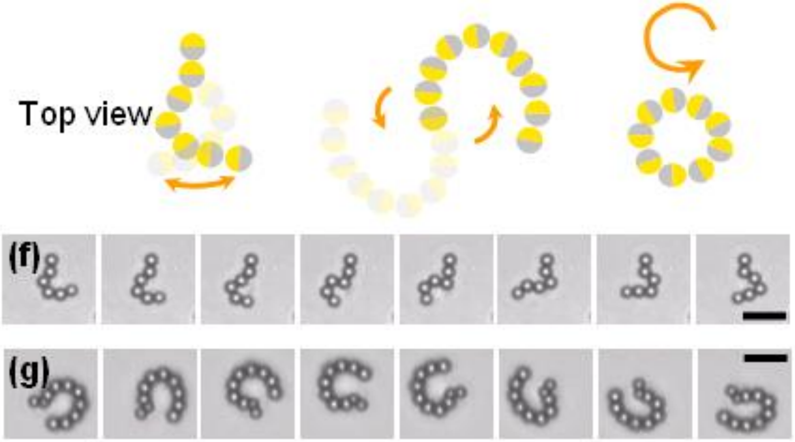


Deformable self-propelled chain



Waving motion

Spiraling motion



Induced Polarization: $P_{eff} = 4\pi a^3 \epsilon_2 Re[K(\omega) E e^{i\omega t}]$

$$K(\omega) = \frac{\epsilon_1 - i\frac{\sigma_1}{\omega} - \epsilon_2 + i\frac{\sigma_2}{\omega}}{\epsilon_1 - i\frac{\sigma_1}{\omega} + 2\epsilon_2 - i\frac{\sigma_2}{\omega}} = \frac{\sigma_1 - \sigma_2}{\sigma_1 + 2\sigma_2}\left(\frac{i\omega\tau_0 + 1}{i\omega\tau_{MW} + 1}\right),$$

$$\tau_0 = \frac{\epsilon_1 - \epsilon_2}{\sigma_1 - \sigma_2}$$

$$\tau_{MW} = \frac{\epsilon_1 + 2\epsilon_2}{\sigma_1 + 2\sigma_2}$$

Electric charging time

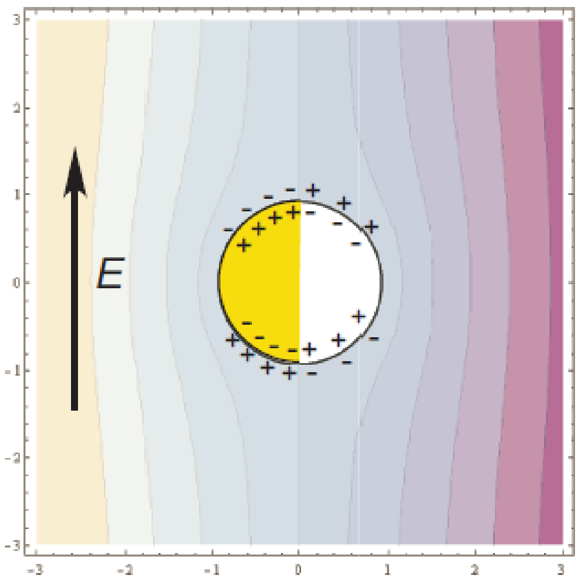


FIG. 5: $f \ll f_c$:

Counter ions in double layer fully charged.

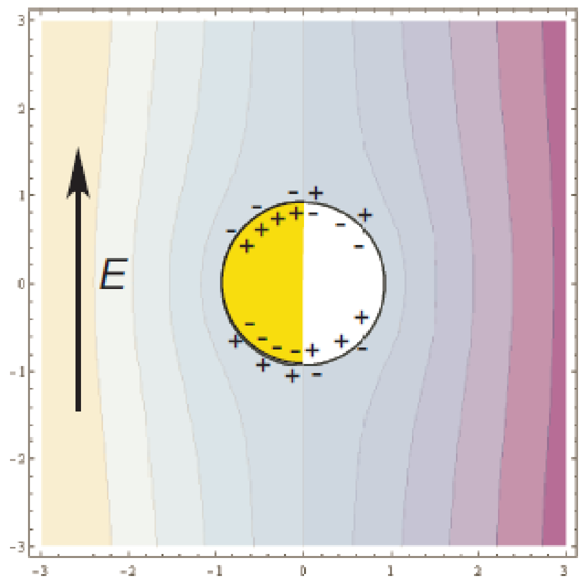


FIG. 6: $f \leq f_c$:

Double layer partially charged.

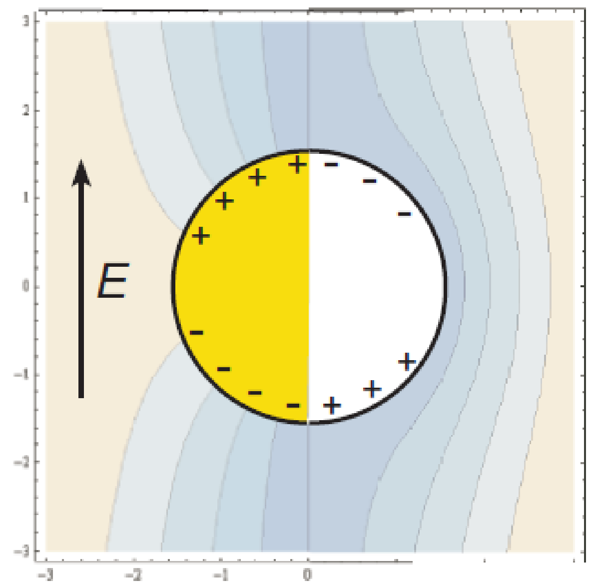
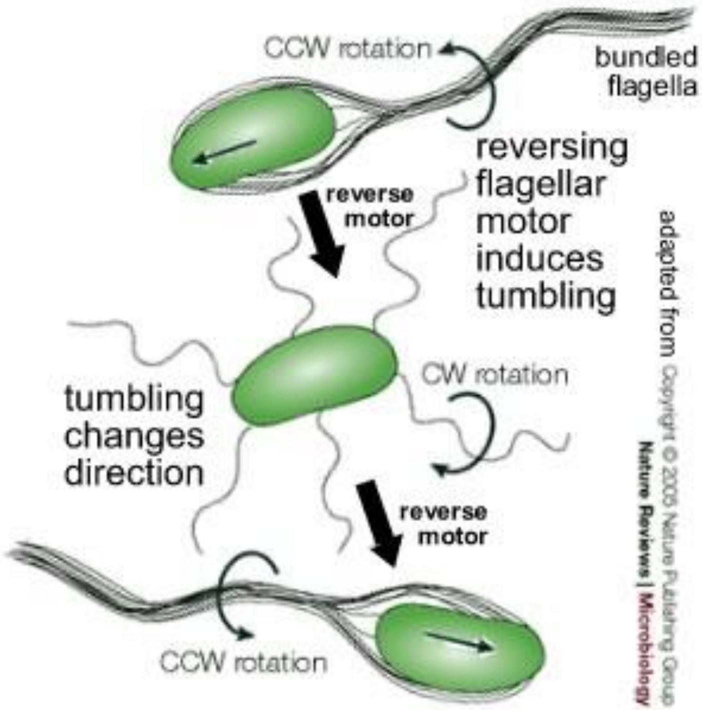
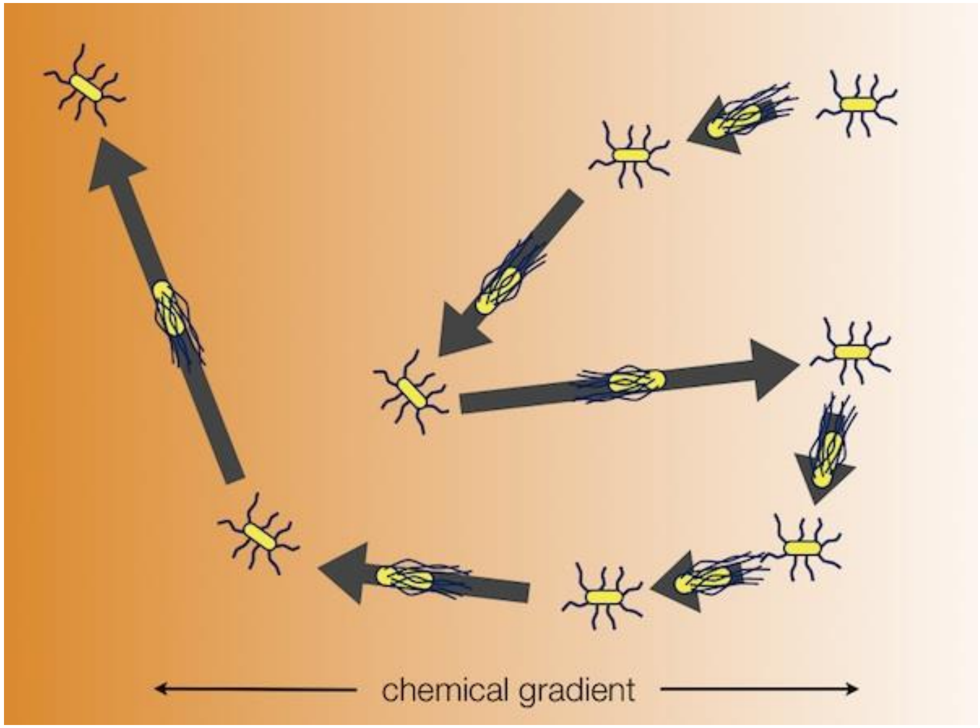


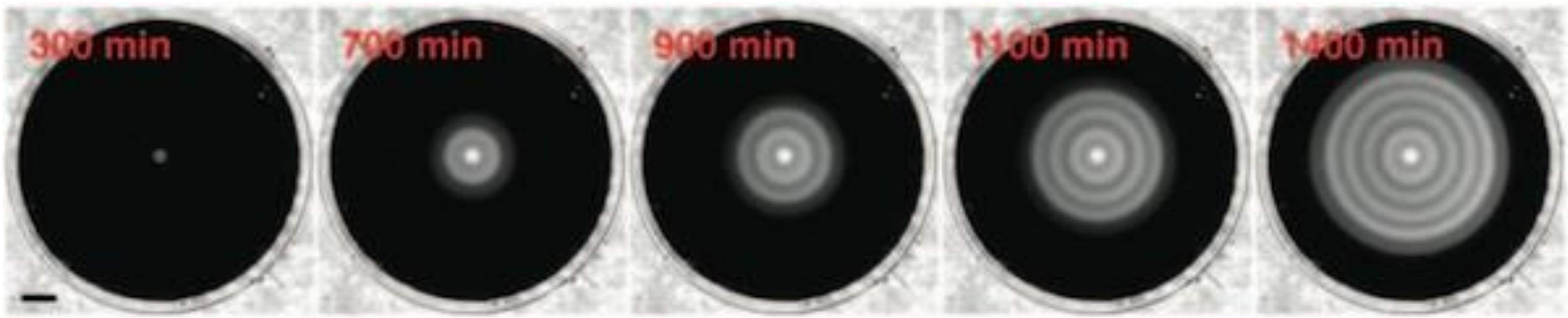
FIG. 7: $f \gg f_c$:

Double layer not charged.

Chemotaxis of Bacteria







http://www.sciencemag.org/content/334/6053/238.abstract

Information and Feedback in Different Systems

	Fluctuation	Information	Feedback	Outcome
Maxwell's demon	Thermal	Speed, position	Biased Choice of fluctuations	Gain Free Energy
Active Particle	Thermal	-------	--------	Enhanced Diffusion
Bacteria (*Escherichia coli*)	tumbling	Chemotactic Signal	Change tumbling freq.	Chemotaxis
Amoeboid cell (*Dictyostelium Discoideum*)	Instability of cell shape	Chemotactic Signal	Biased Choice of random protrusion	Chemotaxis

Summary

- Information thermodynamics can be tested and demonstrated in colloidal systems.
- Different kinds of phoresis can be used to create self-propelled particles and control interaction of particles.

From Brownian to Driven and Active Dynamics of Colloids: Energetics and Fluctuations Part II

Masaki Sano*
Shoichi Toyabe[1]
Hong-ren Jiang[2]
Ryo Suzuki*[3]
*The University of Tokyo
[1]Ludwig Maximilians University Munich
[2]National Taiwan University
[3]Techinical University of Munich

Outline of Part II

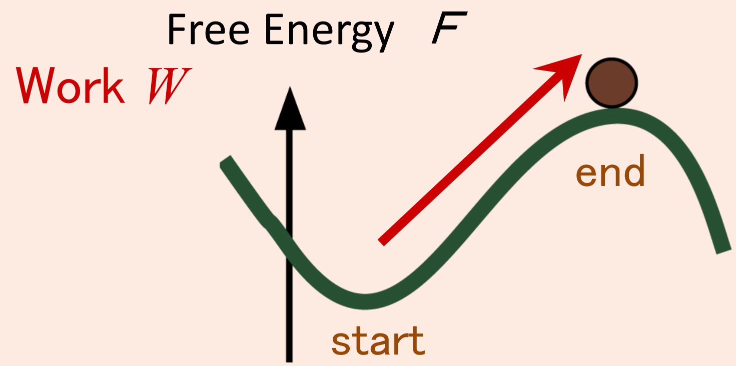
- Information Feedback Control and Generalized Jarzynski Equality

- Introduction to Active particle, Active Matter

Jarzynski Equality

Importance of fluctuations

2 nd law of thermodynamics and FDT can be derived from JE



2 nd law of Thermodynamics

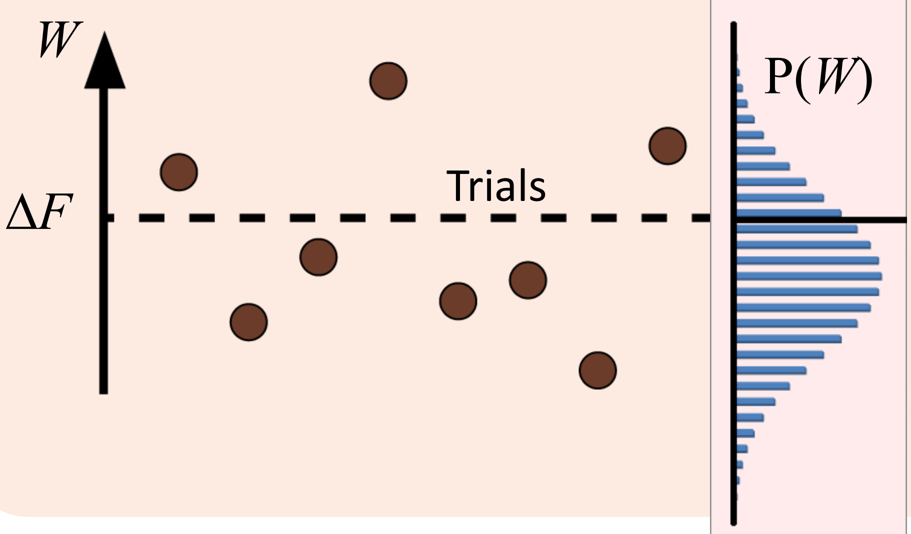
$$W \geq \Delta F = F_2 - F_1$$

Maximum work

W :Work exerted to the system

ΔF :Free energy gain of the system

Small system large fluctuation



Jarzynski equality (1997)

$$\overline{\exp[W/kT]} = \exp[\Delta F/kT]$$

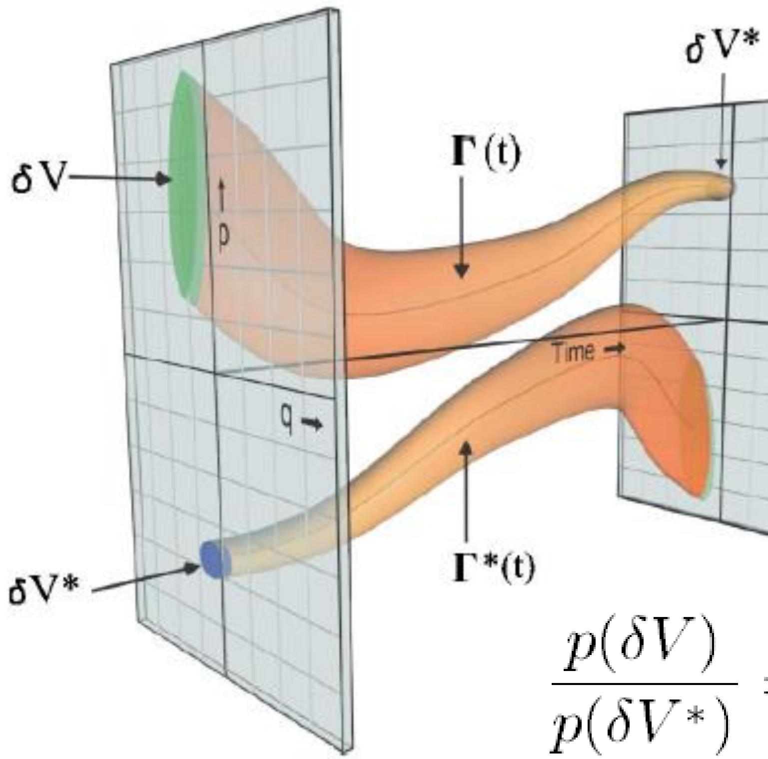
New Theories in Statistical Mechanics

Fluctuation Theorem

- Second law of thermodynamics
- Fluctuation dissipation theorem

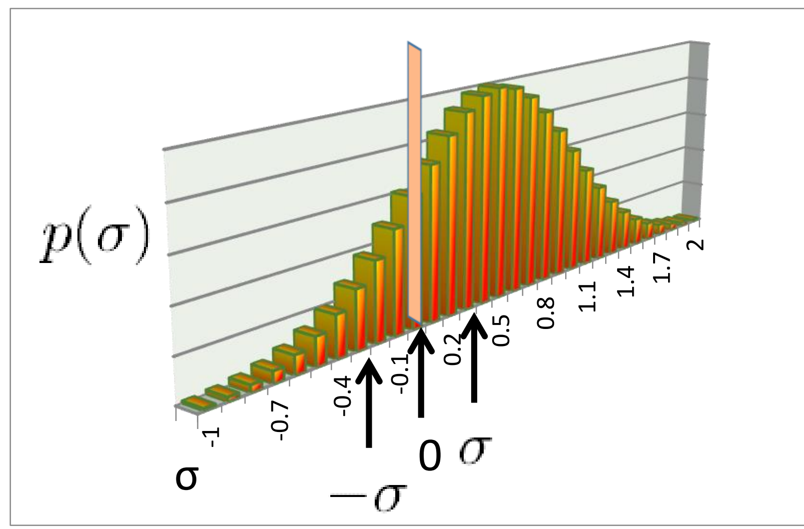
$$\frac{p(\sigma)}{p(-\sigma)} = e^{\sigma}$$

Evans, 1993
Gallavotti, Cohen



$$\frac{p(\delta V)}{p(\delta V^*)} = e^{\dot{\sigma} t}$$

Entropy production: $\sigma = \beta(W - \Delta F)$



Initial conditions producing positive entropy production is much more frequent than the negative entropy production

Confirmed for driven Brownian particles, electric current, etc.

$$\frac{p(\sigma)}{p(-\sigma)} = e^{\sigma}$$

Fluctuation Theorem → **Jarzynski equality**

W :Work performed to the system

F :Free energy gain of the system

$$\langle e^{-(W-\Delta F)/k_BT} \rangle = 1$$

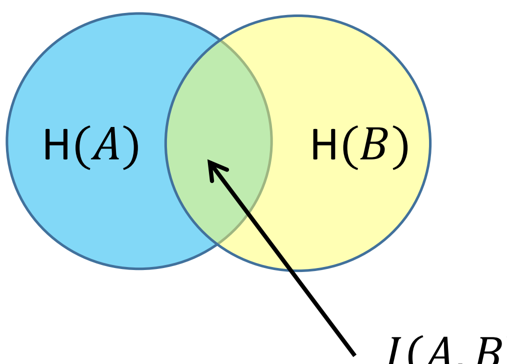
$$e^{-x} \geq 1 - x$$

2nd law of thermodynamics: $\langle W \rangle - \Delta F \geq 0$

Generalized Jarzynski equality including information

$$\langle W \rangle \geq \Delta F - kT\langle I \rangle$$

I : mutual information
measurement and control have errors



Correspondingly generalized Jarzynski equality:

$$\Rightarrow \langle e^{(\Delta F - W)/k_BT} \rangle = \gamma$$

Sagawa & Ueda, PRL (2010)

Maxwell's demon

- Violation of the second law of thermodynamics

(1871)

James Clerk Maxwell (1831-1879)

Opening & closing door do not perform work to atoms.

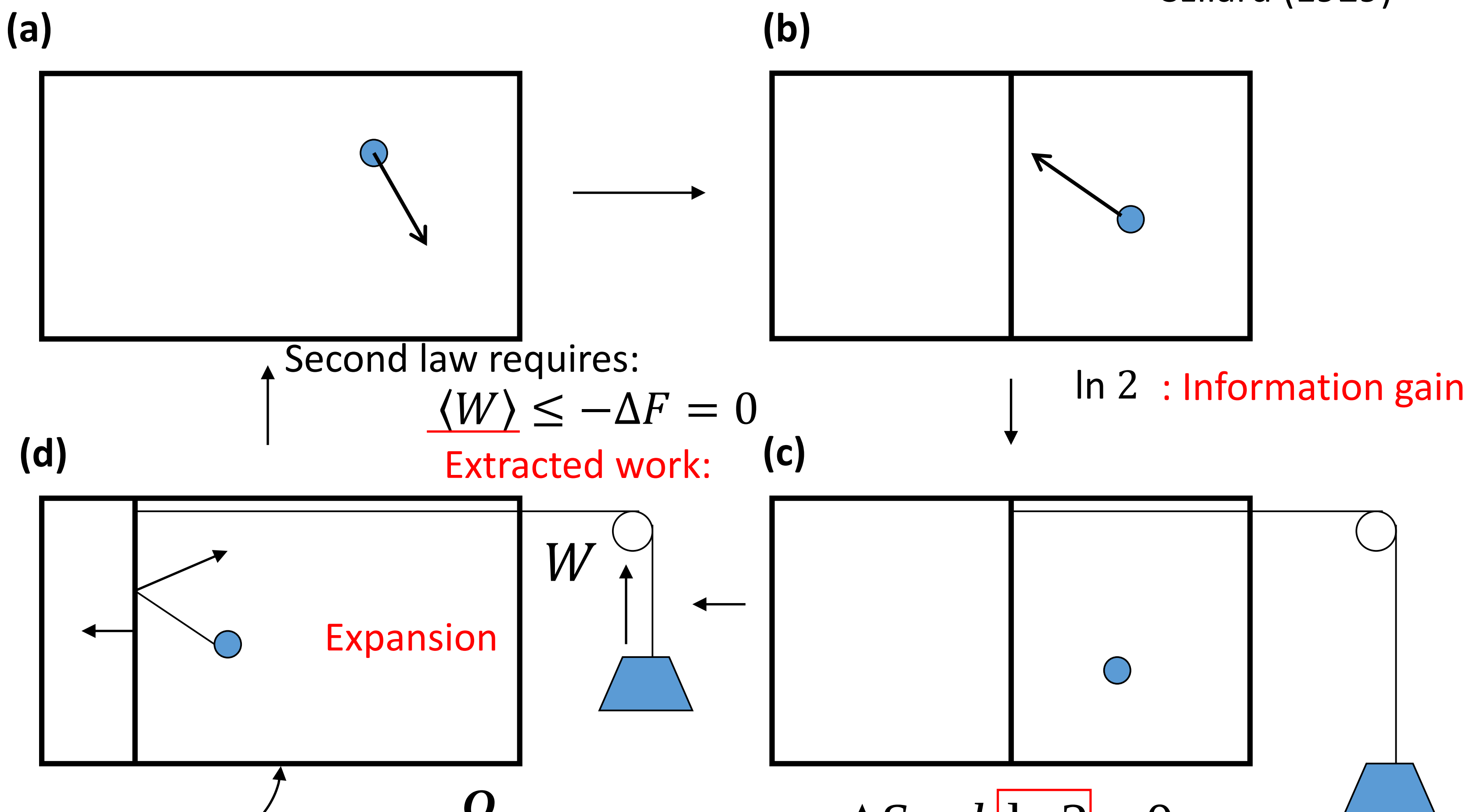
⇒ 2nd law really violate？

⇒ controversial state lasted more than 150 years.

Maxwell's Demon and Szilard Engine

The simplest and analyzable Maxwell's demon

Szilard (1929)

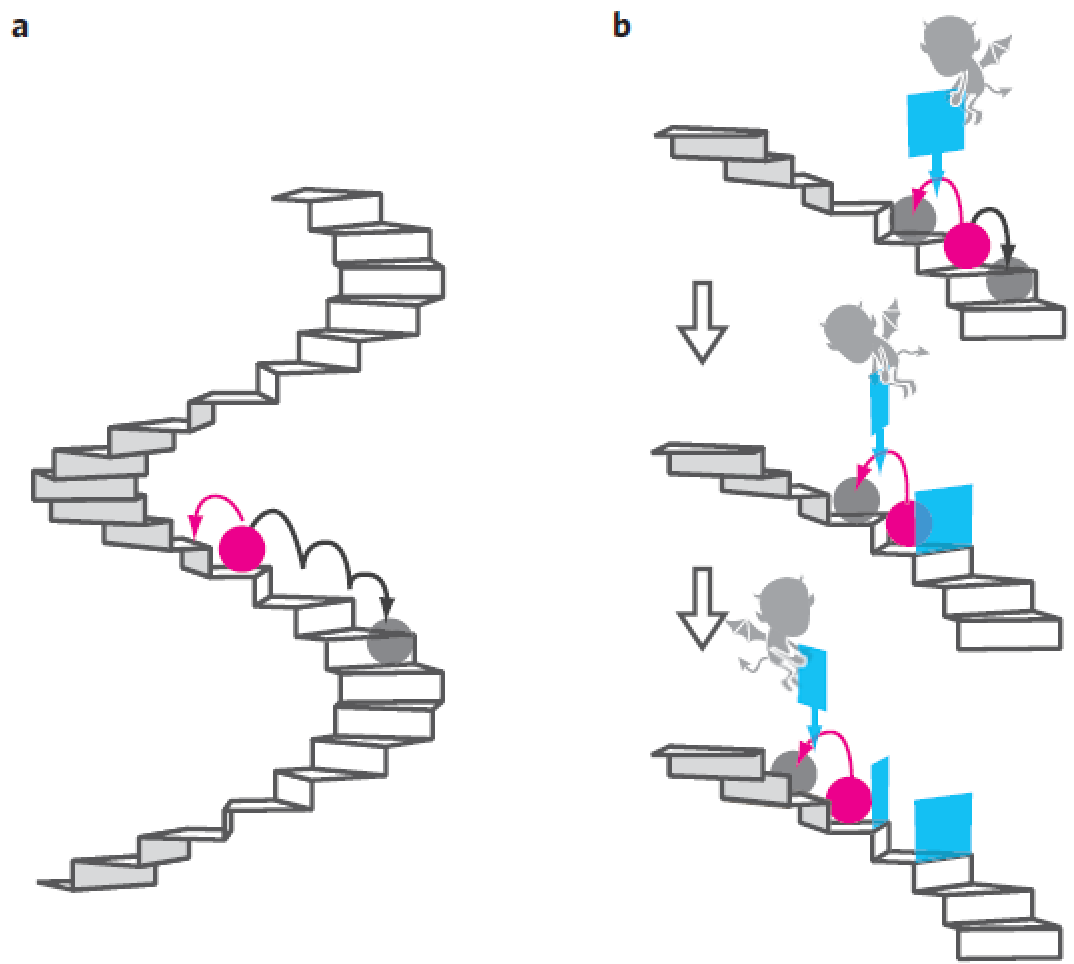


$\boldsymbol{Q} = \boldsymbol{W}$ $\Delta \boldsymbol{S} = -\dfrac{\boldsymbol{Q}}{\boldsymbol{T}} < 0$ Heat Bath $\Delta S = k \boxed{\ln 2} > 0$

Entropy gain of gas

However we gain, $W = k_B T \boxed{\ln 2}$: Information gain → decrease of entropy

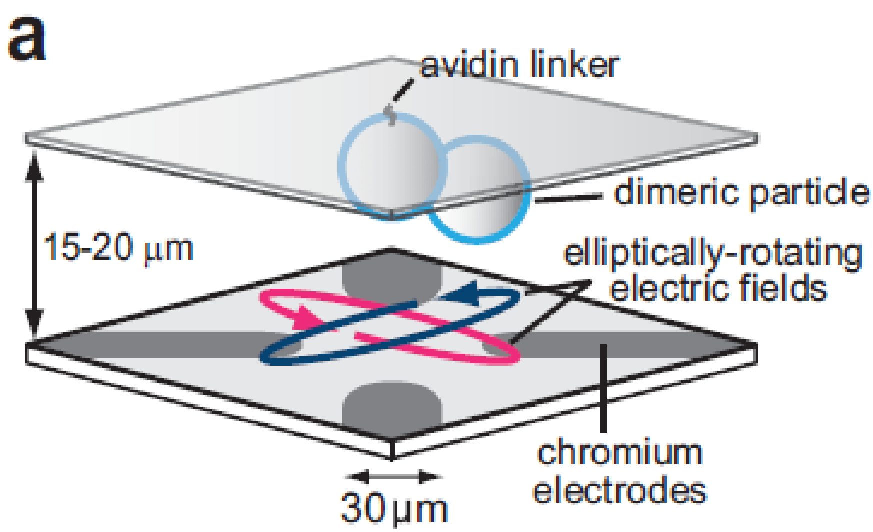
Schematic illustration of the experiment

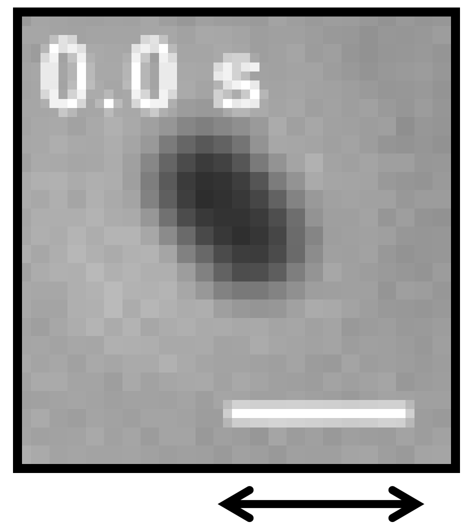


Toyabe, Sagawa, Ueda, Muneyuki, Sano, Nature Physics, 6**,** 988**,** (2010))

Experimental Setup

- Dimeric polystyrene particle (300nm) is linked on the substrate with a biotin.
- Particles exhibit a rotational Brownian motion.

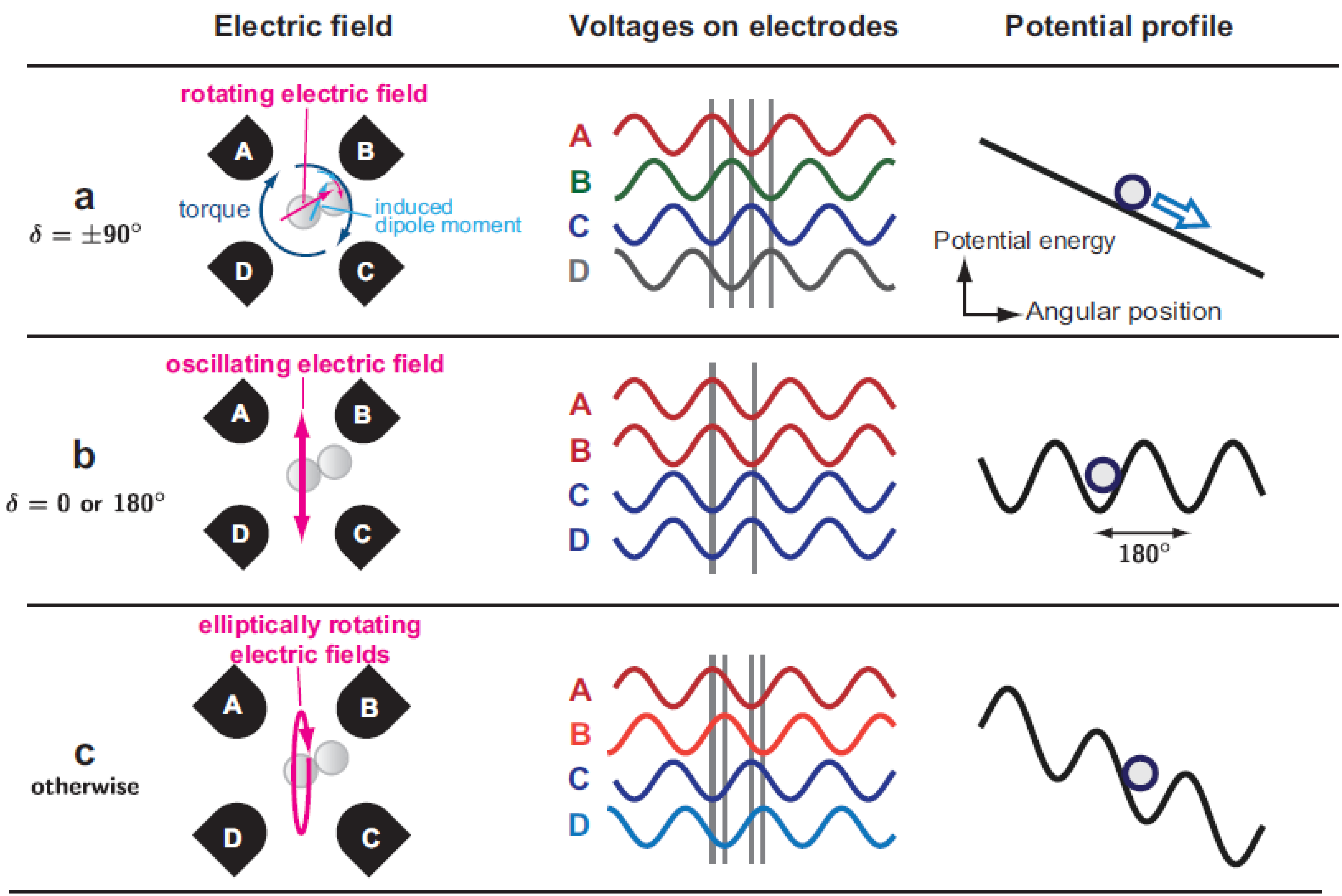




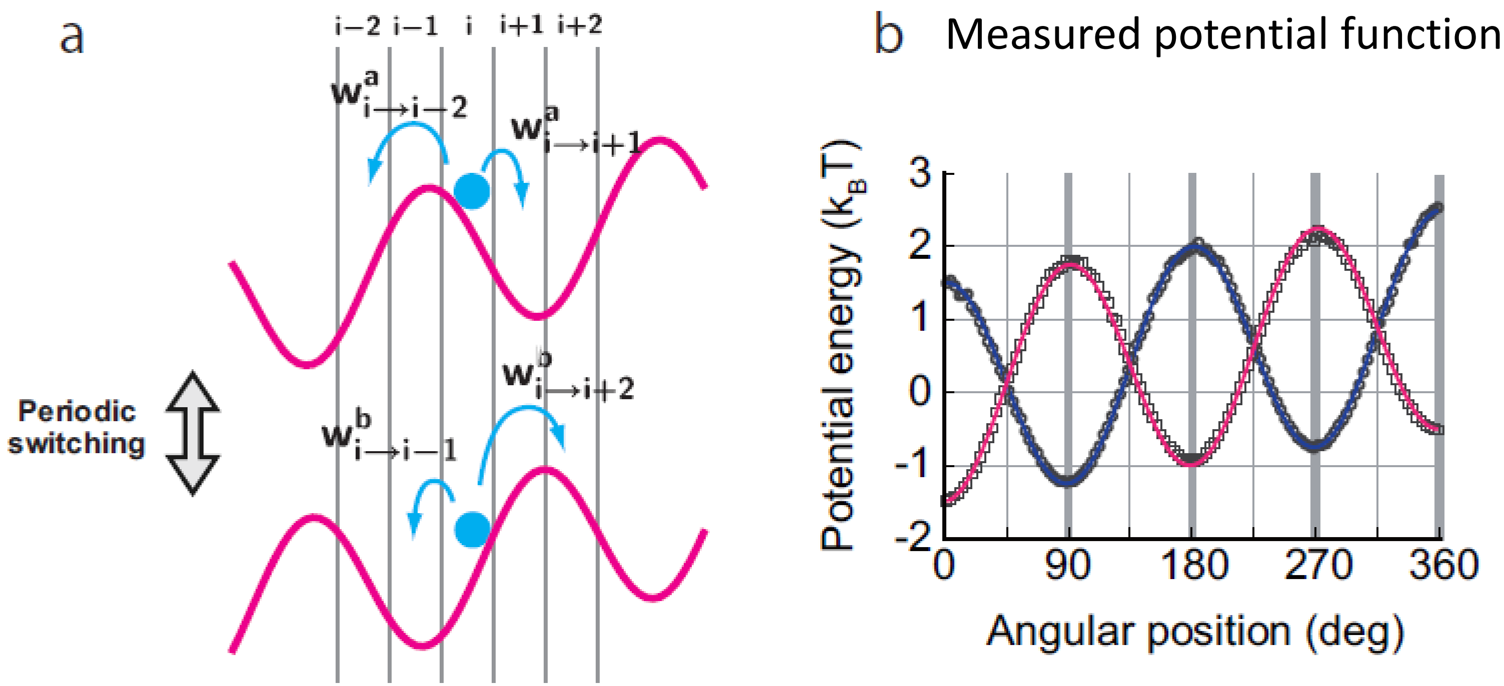
1 μm (1/1000 mm)

- Quadrant electrodes are patterned on the substrate

How to produce a spiral-stair-like potential



Estimating a potential function from the data

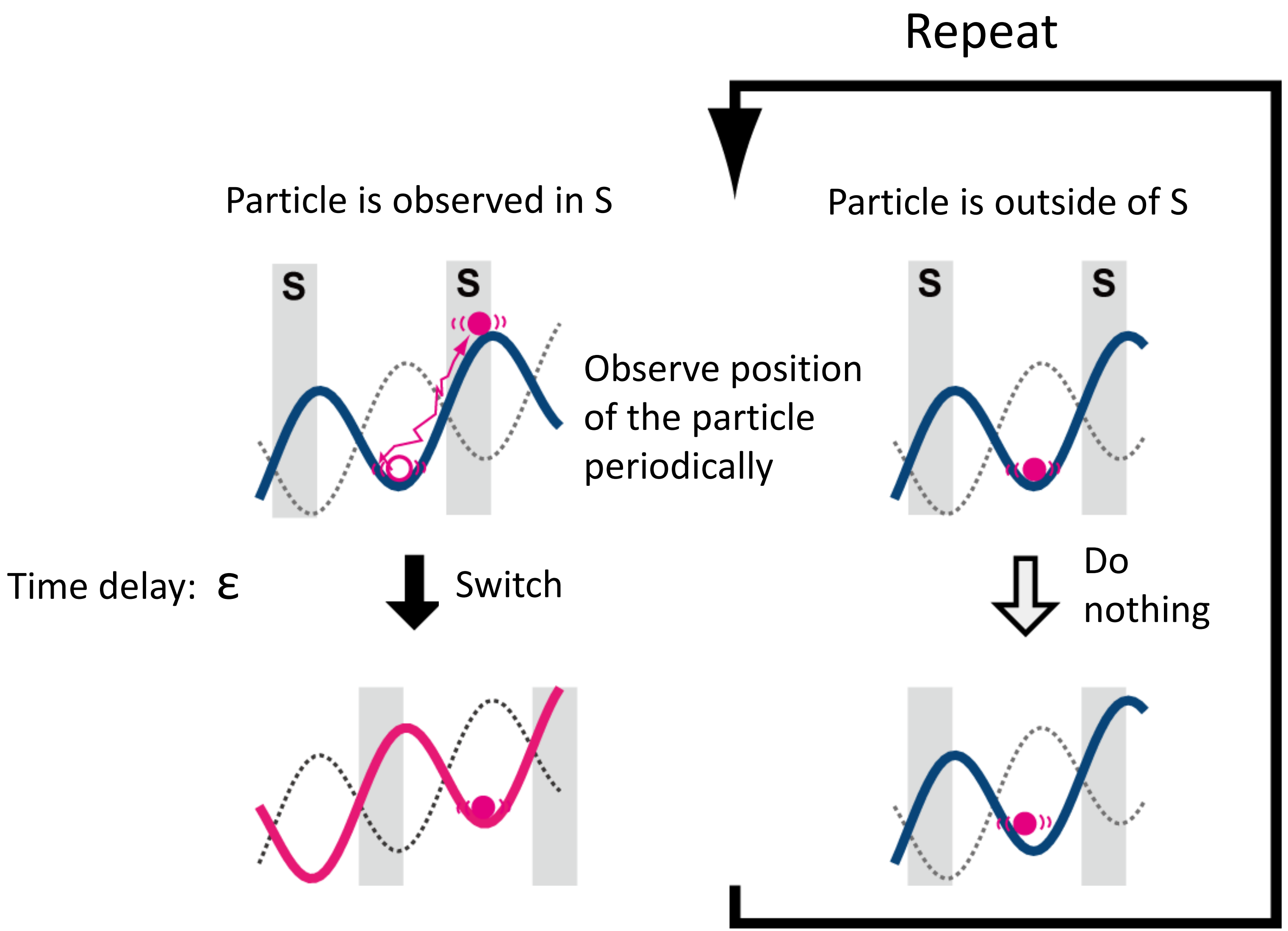


$$\varepsilon^2(\{U_i\}) \equiv \sum_{i<j} \sqrt{n_{i\to j} n_{j\to i}} \left[\Delta U_{i\to j} - \Delta U'_{i\to j}\right]^2,$$

where $\Delta U'_{i\to j} \equiv k_B T \left[\ln w_{j\to i} - \ln w_{i\to j}\right]$

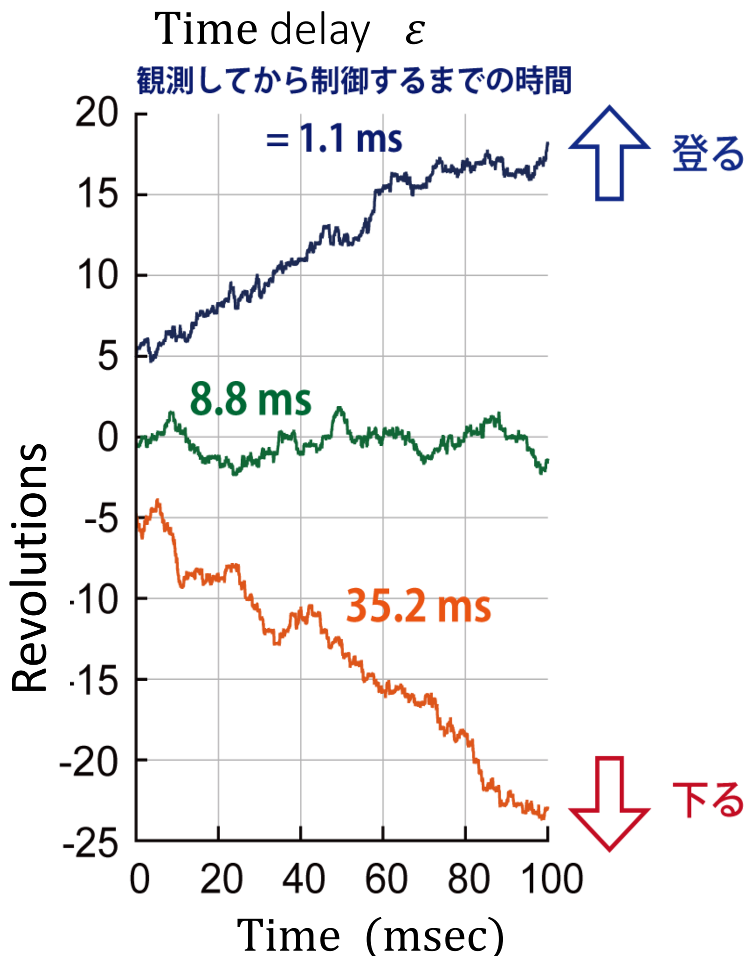
Minimize ε^2

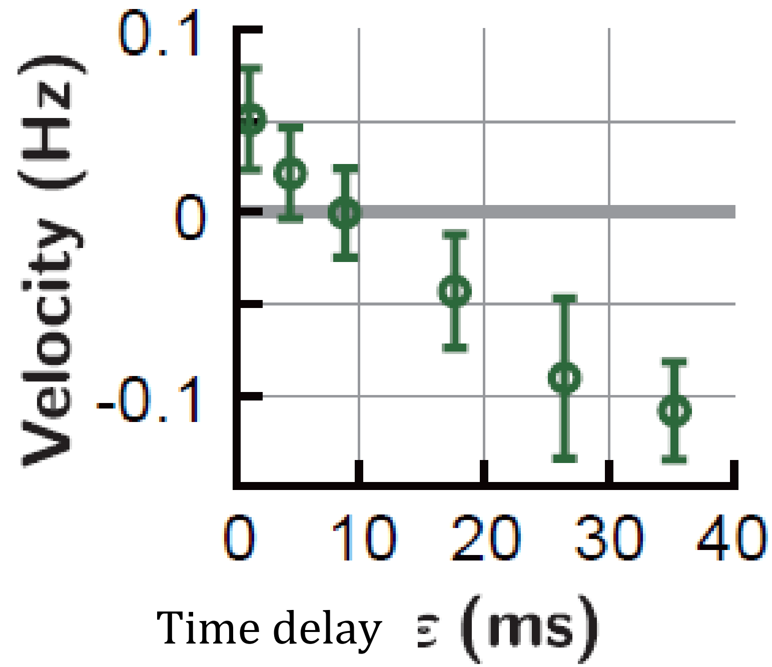
Feedback control based on information contents

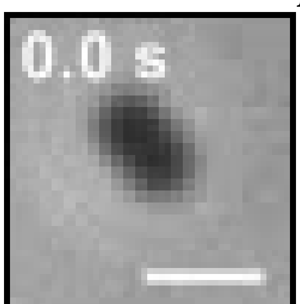


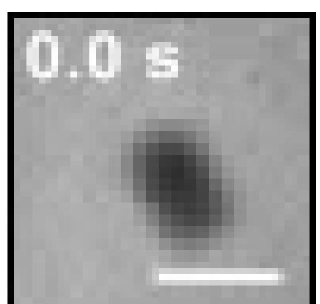
Feedback by Information

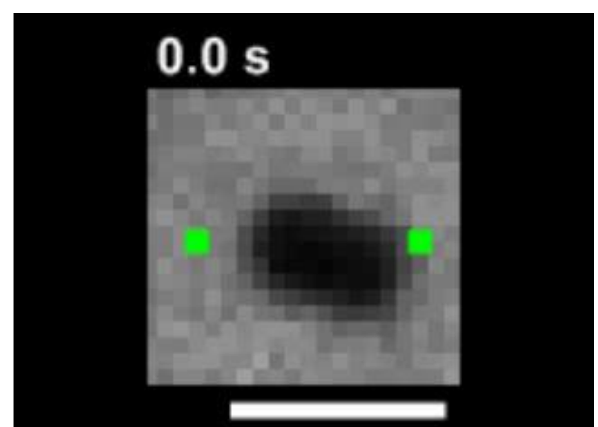
Trajectories under feedback control





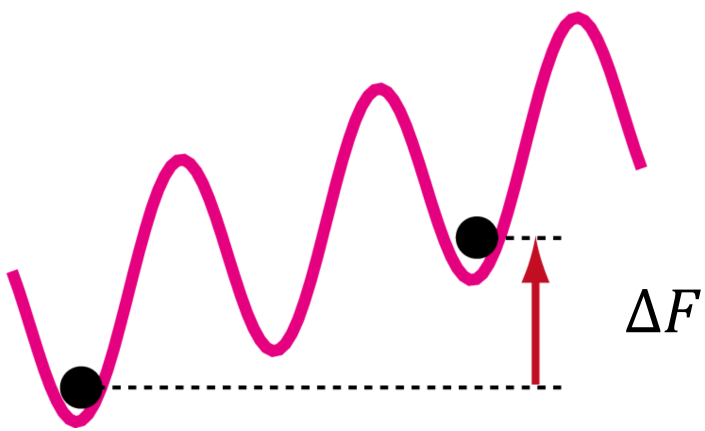






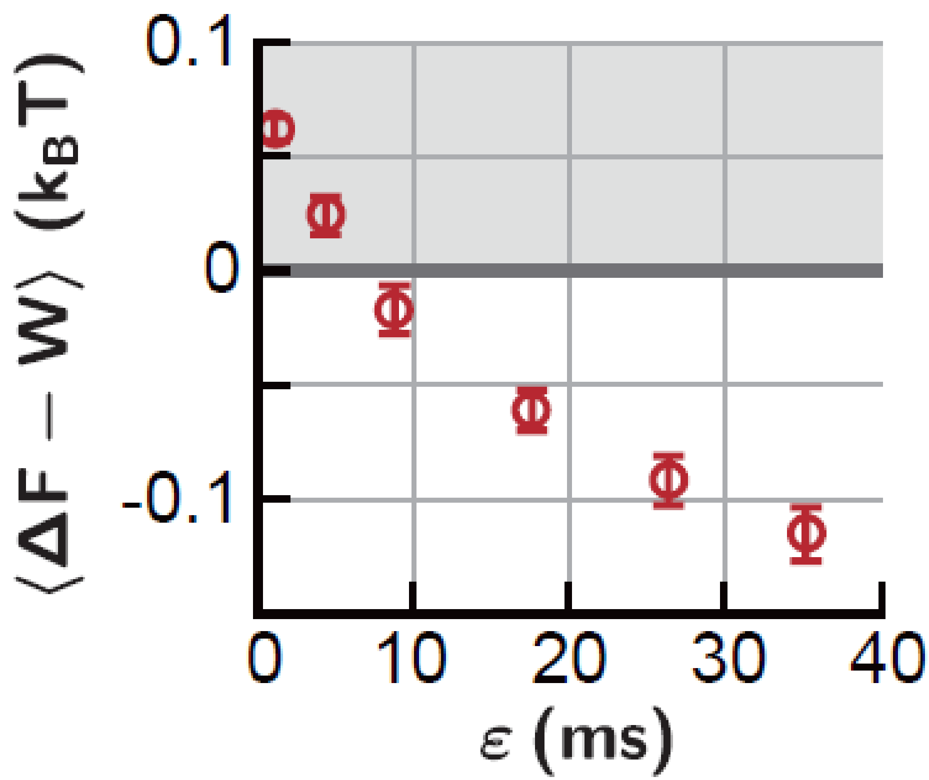
Calculation of Free Energy

- Free energy gain

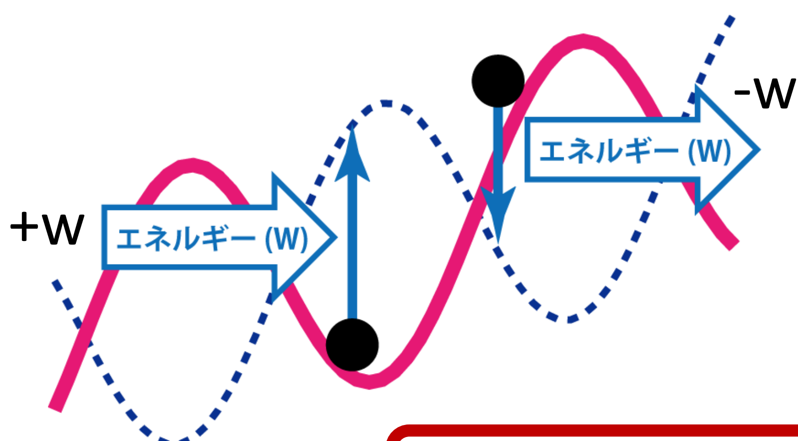


W :Work performed to the system

F :Free energy gain of the system



- Work done to the particle

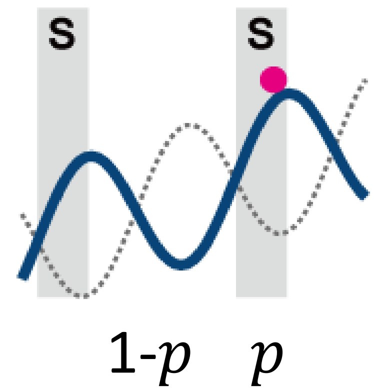


Calculate（ΔF-W）

Efficiency of Information-Energy Conversion

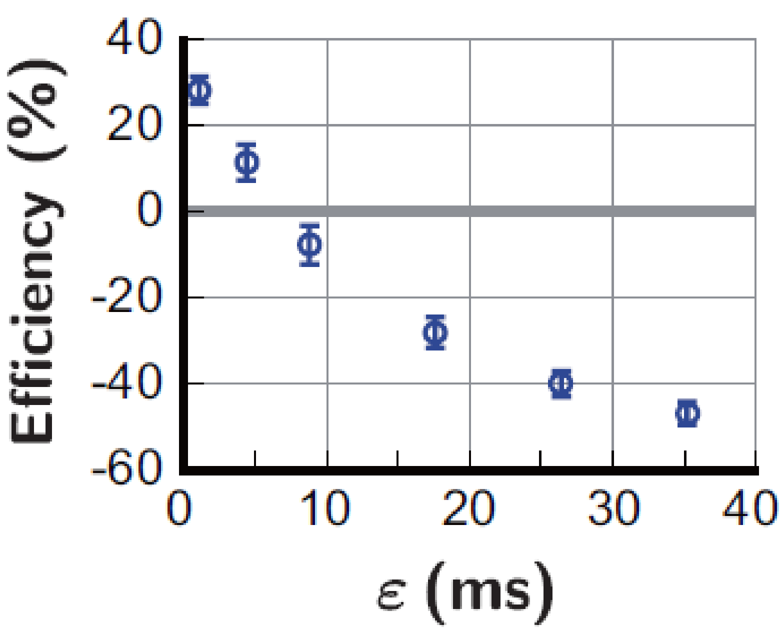
Information gained by the observation:

$$I = -p \ln p \; - (1-p)\ln((1-p)$$



Efficiency of Information-Energy Conversion :

$$\vartheta = \frac{\Delta F - W}{k_B T I}$$



28% Efficiency

Experimental test of generalized Jarzynski equality

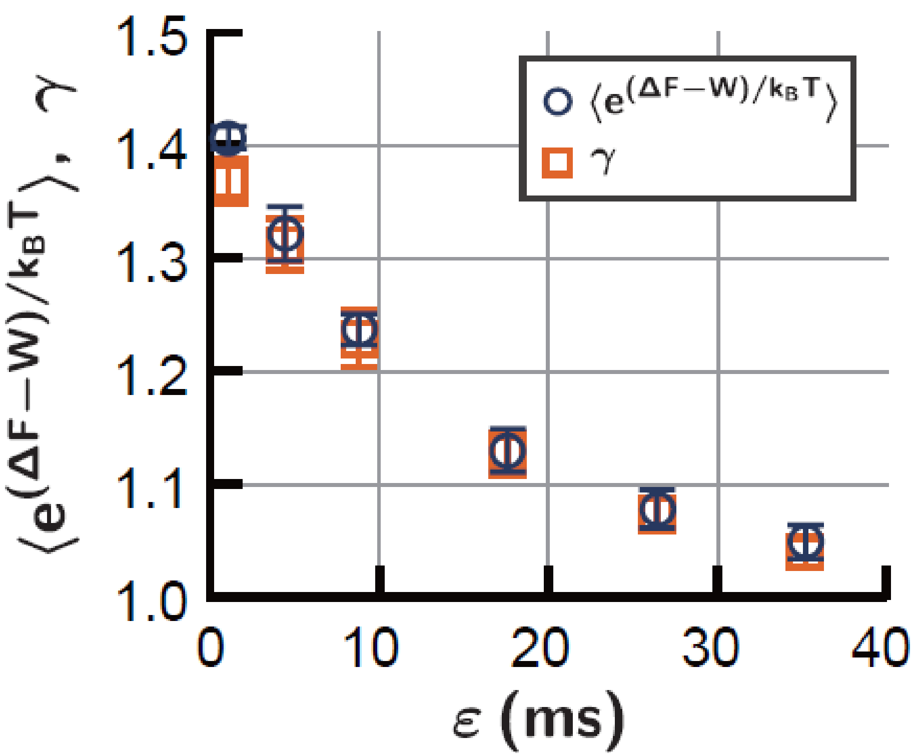
- Entropy Production

$$\left\langle e^{\beta(\Delta F - W)} \right\rangle = \gamma$$

generalized Jarzynski equality
Sagawa, Ueda, PRL (2010)

Feedback Efficacy

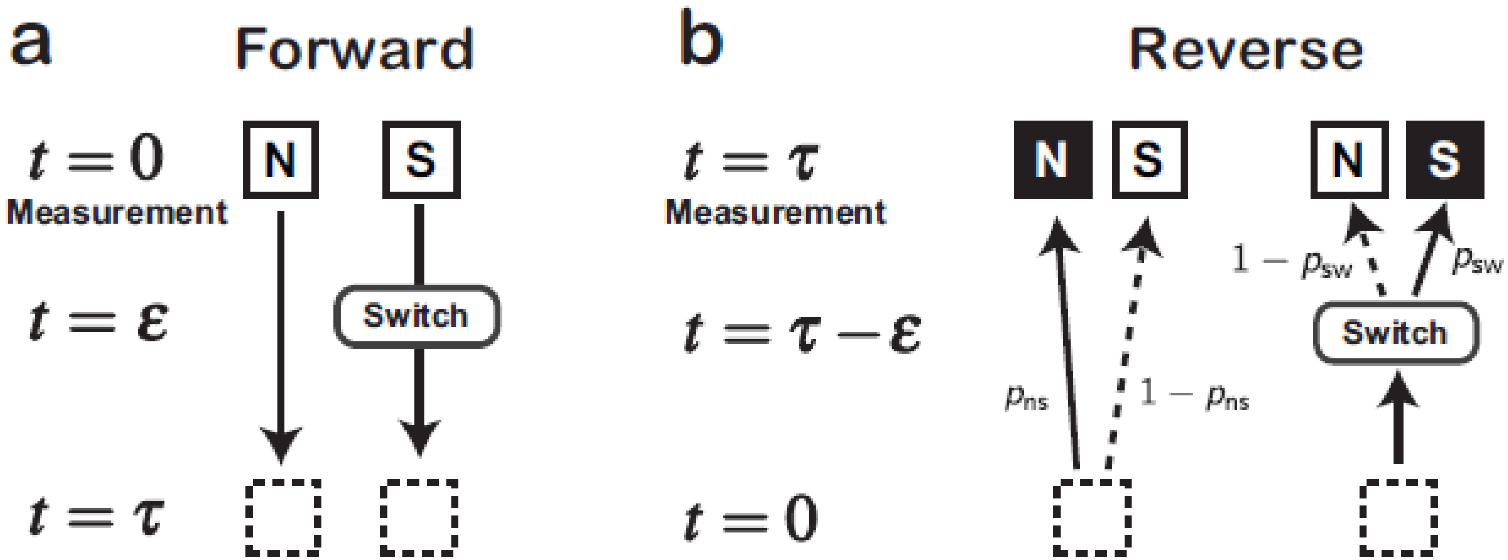
- **A fundamental principle to relate energy and information feedback**



Agrees within measurement accuracy

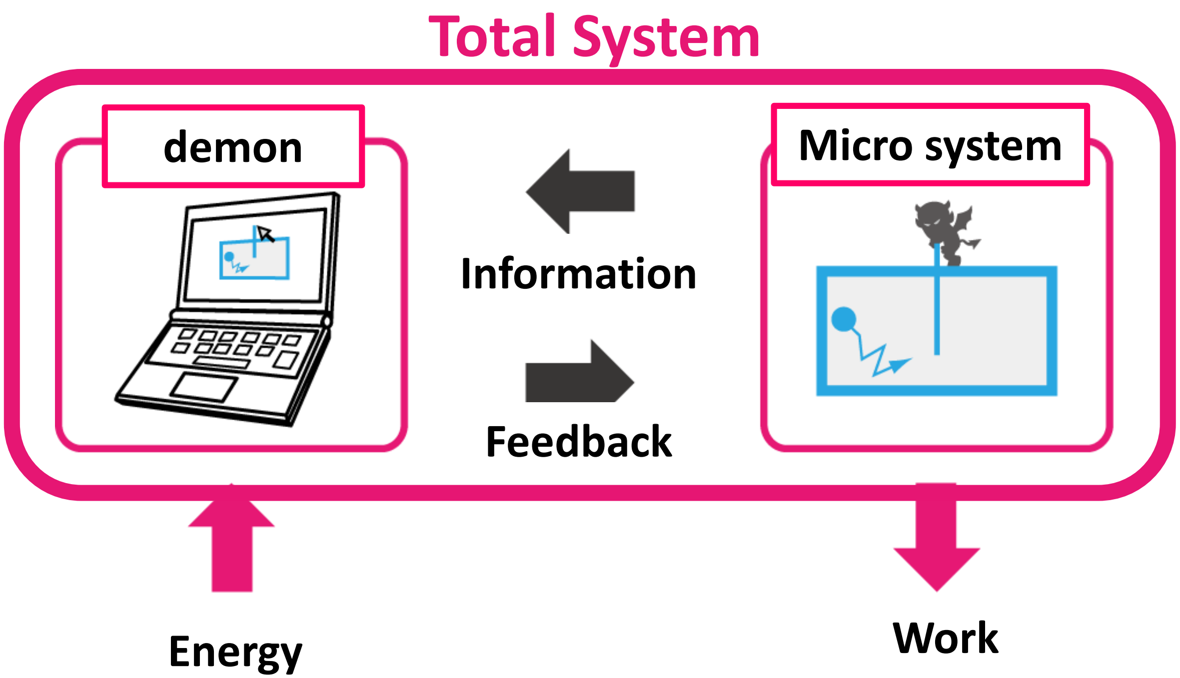
Toyabe, Sagawa, Ueda, Muneyuki, Sano, Nature Physics, 6, 988, (2010))

How to measure the feedback efficacy



$$\gamma = p_{sw} + p_{ns} \leq\geq 1$$

Consistency with 2^{nd} Law



- 2^{nd} law holds for the total system.
- Information-energy conversion is realized when we look at the small system.

Self-propelled dynamics of Janus particle

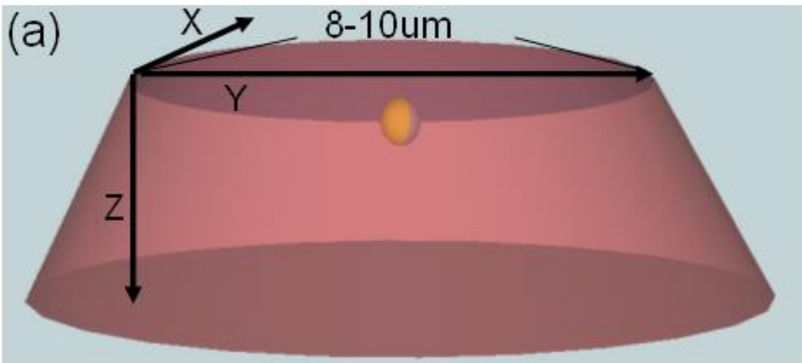
- By a local temperature gradient:

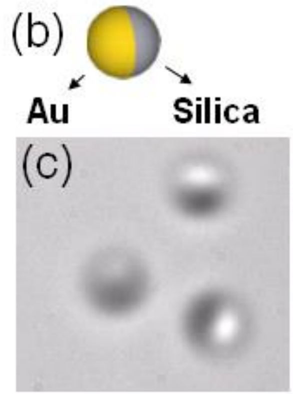
  Self-Thermophoresis

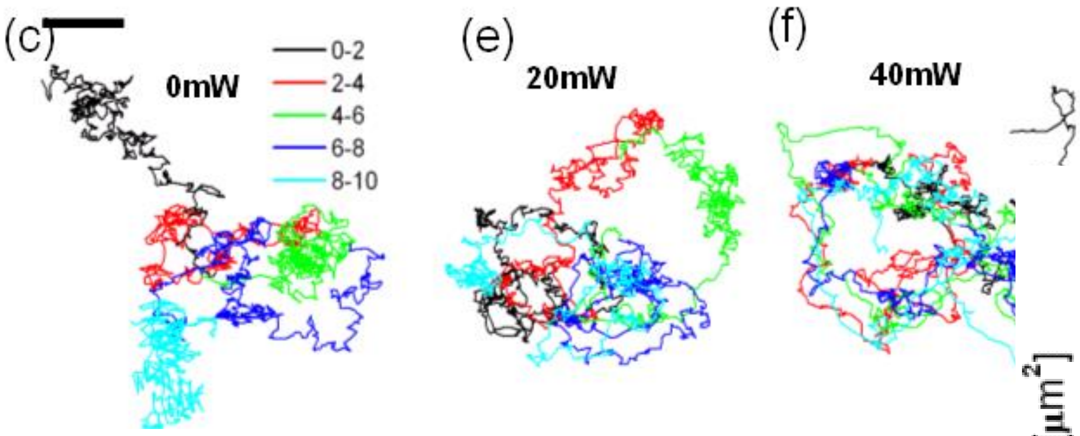
- By electric field:

  Induced Charge Electro-Osmosis (ICEO)

Self-propulsion of Janus particle I : Temperature field



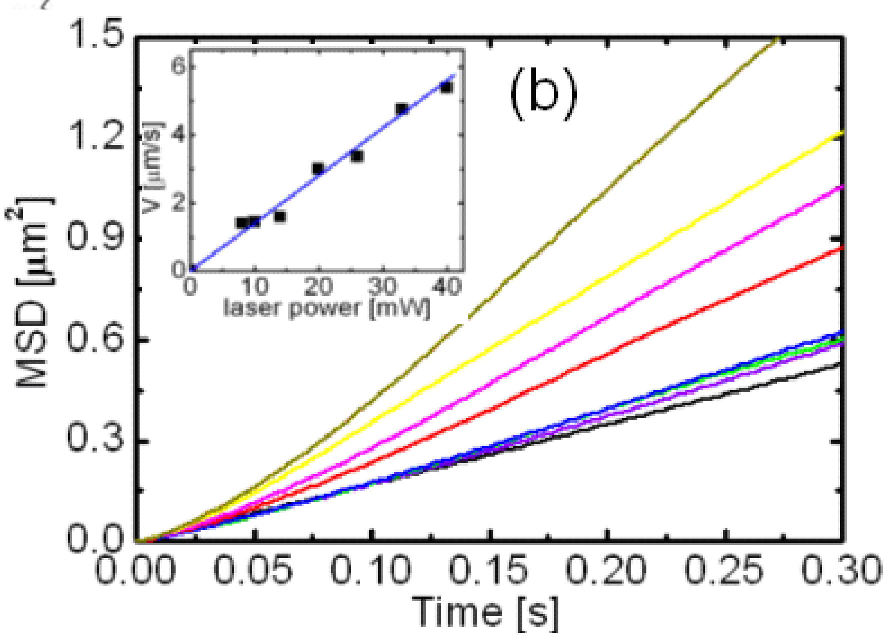




$$\left\langle \Delta r^2(t) \right\rangle = 2\tau V^2[t - \tau(1 - e^{-t/\tau})]$$

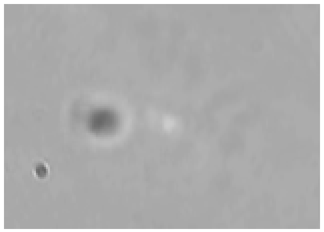
$$\left\langle \Delta r^2(t) \right\rangle \sim V^2 t^2, \qquad t << \tau$$

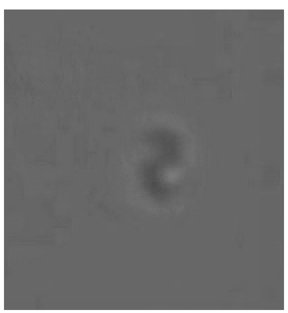
$$\left\langle \Delta r^2(t) \right\rangle \sim \tau V^2 t, \qquad t >> \tau$$

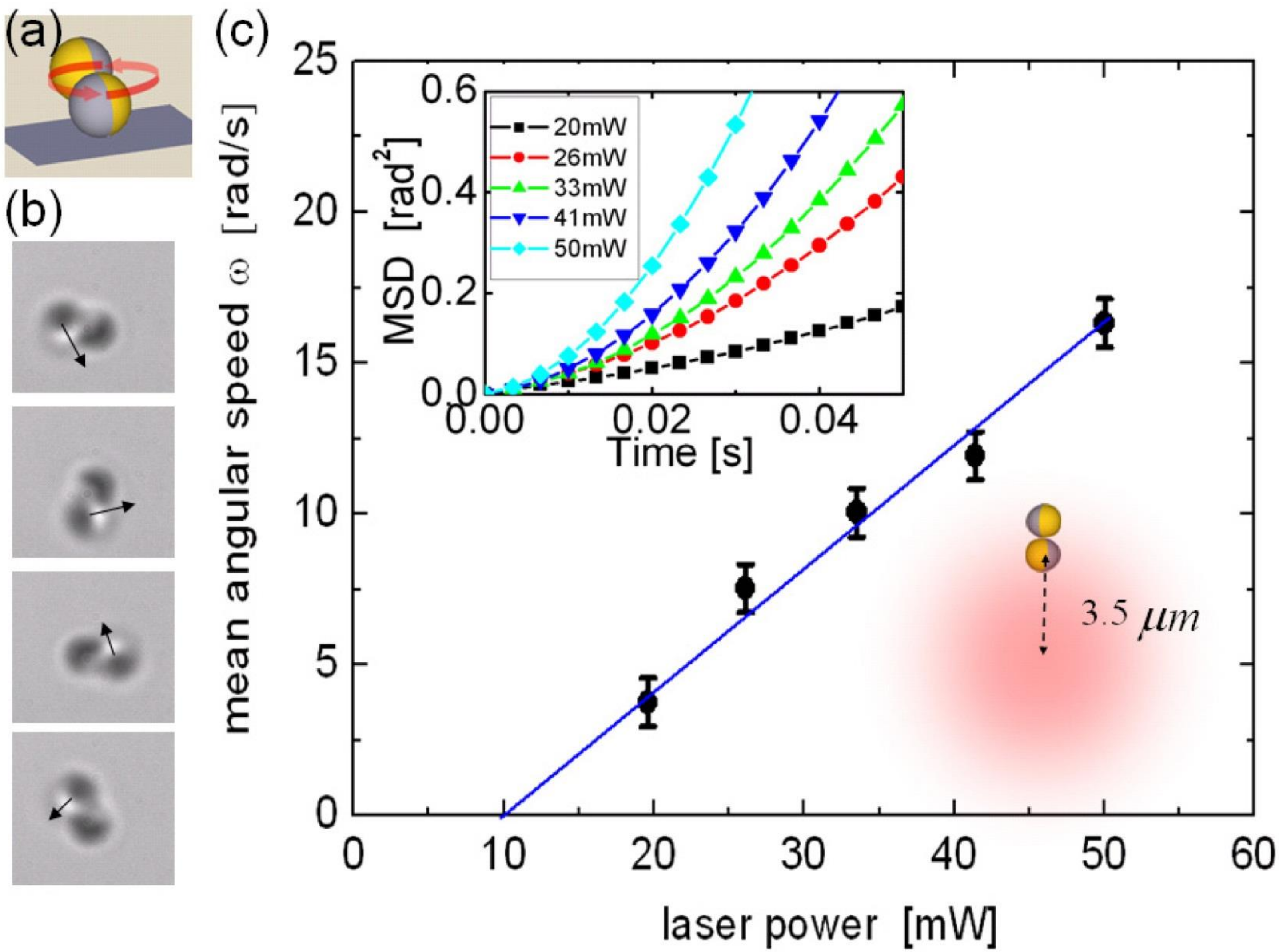


HR Jiang, Yoshinaga, Sano, PRL, 105, 268302 (2010)

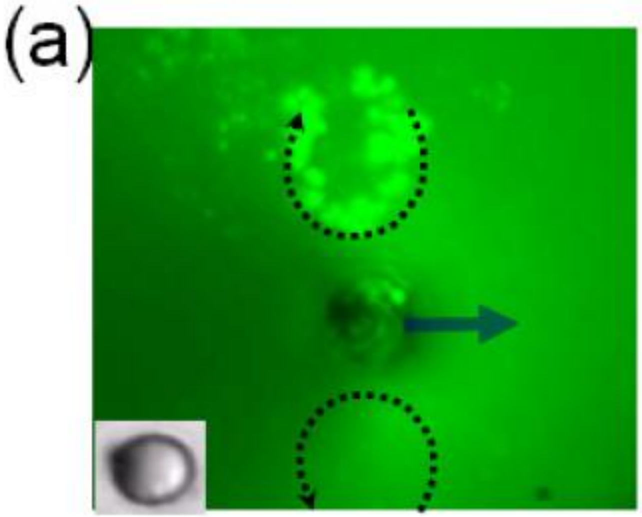
Rotation of Chiral Doublet: Thermophoresis





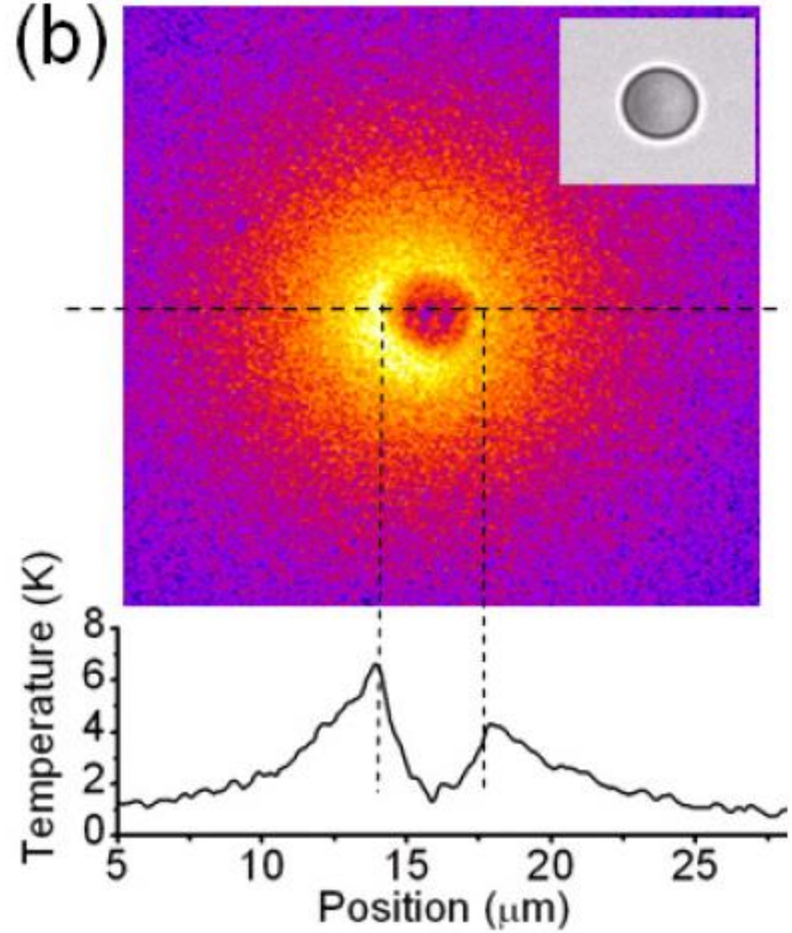


HR Jiang, Yoshinaga, Sano, PRL (2010)



Induced flow visualized by tracer particles around the fixed Janus particle

Temperature distribution around a Janus Particle



Temperature distribution :

$$T(R) = T_\infty + \sum_{n=1}^{\infty} \frac{q_n R}{(n+1)\kappa_o + n\kappa_i} P_n(\cos\theta).$$

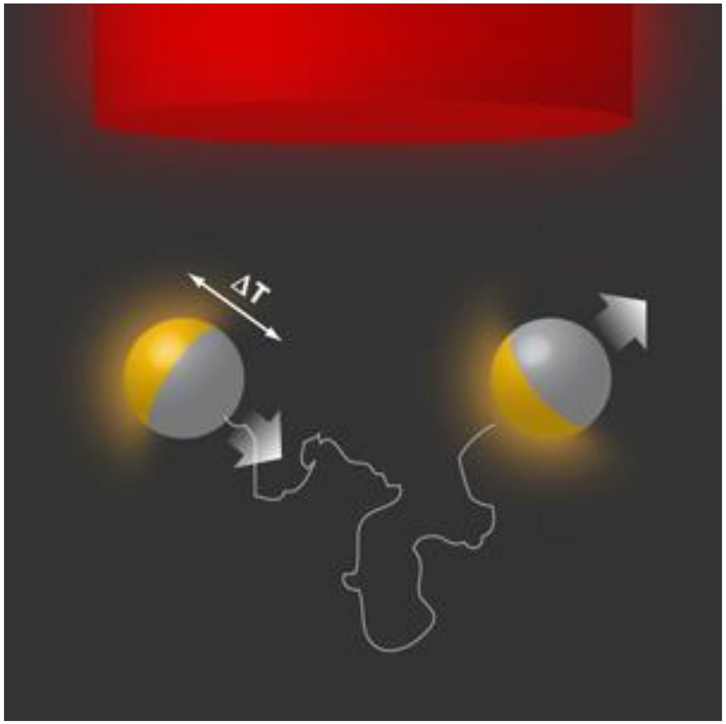
$$q(\theta) \;=\; \kappa \mathbf{e}_n \cdot \nabla T$$

$$\Delta T = 3\epsilon I R / 2(2\kappa_o + \kappa_i)$$

Seed of self-thermophoresis of Janus particle is related to the response to the external gradient

$$V = -\frac{1}{4} D (S_T^0 + S_T^G) \frac{\epsilon I}{2\kappa_o + \kappa_i}.$$

Self-thermophoresis



Viewpoint in Physics, PRL (2010)

Light is absorbed on the metal side:

$$-\kappa_o \mathbf{n} \cdot \nabla T_o + \kappa_i \mathbf{n} \cdot \nabla T_i = q(\theta).$$

$$T(R) = T_0 + \sum_{n=0}^{\infty} \frac{q_n R}{(n+1)\kappa_o + n\kappa_i} P_n(\cos\theta)$$

Theoretical calculation by N. Yoshinaga

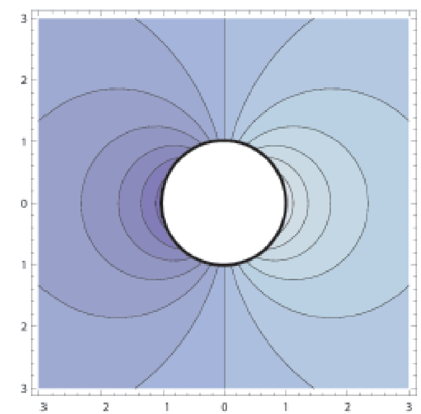
Effective slip velocity: $\boldsymbol{v}_s = v_s \boldsymbol{e}_\theta = \mu |\nabla T|_s \boldsymbol{e}_\theta$

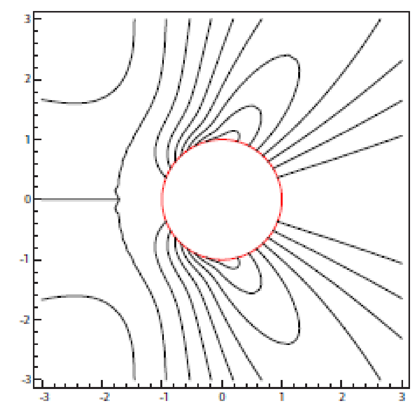
$$\mu = -(k_B/\eta)\Gamma\lambda$$

Characteristic length: $\lambda = \Gamma^{-1} \int c_0 y (e^{-\beta U_0} - 1) dy$

U_0 : Interaction potential between the surface and fluid

$$\Gamma = \int c_0 (e^{-\beta U_0} - 1) dy \qquad V = -\frac{1}{2} \int_0^\pi v_s \sin^2\theta d\theta.$$





Migration in an uniform external temperature gradient

$$V = -\mu T_1 = -D S_T T_1$$

T_1 : Temperature diff. across the particle

$$\mu = D S_T$$

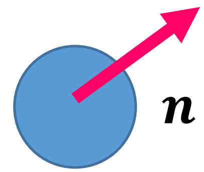
$$V = -\frac{1}{4} D (S_T^0 + S_T^G) \frac{\epsilon I}{2\kappa_o + \kappa_i}.$$

Migration speed is determined by the average of Soret coeff.

Stochastic Dynamics of Active particle

Langevin equation:

$$\gamma \boldsymbol{v} = -\frac{\partial U}{\partial \boldsymbol{r}} + \underline{\beta \boldsymbol{n}} + \boldsymbol{\xi}(t)$$



Fokker-Planck equation:

$$\frac{\partial P(\mathbf{r}, \mathbf{n})}{\partial t} = -\frac{\partial}{\partial \mathbf{r}} \cdot (\mathbf{v}P) - \mathcal{R} \cdot (\omega P)$$

$$\mathcal{R} = \boldsymbol{n} \times \frac{\partial}{\partial \boldsymbol{n}}$$

$$\mathbf{v} = -D\frac{\partial}{\partial \mathbf{r}}(k_B T \ln P + U) + \alpha \mathbf{n}.$$

$$\omega = \frac{1}{\zeta_r}\mathbf{N} = -\frac{1}{\zeta_r}\mathcal{R}(k_B T \ln P + U),$$

$$\frac{\partial P(\mathbf{r}, \mathbf{n}, t)}{\partial t} = \frac{\partial}{\partial \mathbf{r}} \cdot D\left(\frac{\partial P}{\partial \mathbf{r}} + \frac{P}{k_B T}\frac{\partial U}{\partial \mathbf{r}}\right) - \alpha \frac{\partial}{\partial \mathbf{r}} \cdot (\mathbf{n}P) + D_r \mathcal{R} \cdot \left(\mathcal{R}P + \frac{P}{k_B T}\mathcal{R}U\right).$$

$$D = \frac{k_B T}{6\pi\eta a} \qquad D_r = \frac{k_B T}{8\pi\eta a^3}$$

$$U = \frac{1}{2}kr^2 \qquad \mathcal{R}U = 0.$$

Correlation function of the polarity direction:

$$< \mathbf{n}(t) \cdot \mathbf{n}(0) >= \int d\mathbf{n}d\mathbf{n}'d\mathbf{r}d\mathbf{r}' \left[\mathbf{n} \cdot \mathbf{n}' G(\mathbf{n}, \mathbf{n}', t) P_{\text{eq}}(\mathbf{r}', \mathbf{n}') \right]$$

Equilibrium distributionc

$$P_{\text{eq}}(\mathbf{r}, \mathbf{n}) = \frac{1}{4\pi} \left(\frac{k}{2\pi k_B T} \right)^{-3/2} e^{-\frac{k\mathbf{r}^2}{2k_B T}}.$$

Rotational Diffusion:

$$< \mathbf{n}(t) \cdot \mathbf{n}(0) >= \exp(-2D_r t).$$

$$\frac{\partial}{\partial t} < (\mathbf{r}(t) - \mathbf{r}(0))^2 >= \int d\mathbf{r}d\mathbf{n}'d\mathbf{r}d\mathbf{r}' \left[(\mathbf{r} - \mathbf{r}')^2 \frac{\partial G}{\partial t} P_{\text{eq}} \right].$$

MSD of active particle:

$$< (\mathbf{r}(t) - \mathbf{r}(0))^2 >= \frac{6k_B T}{k} + \frac{2\alpha^2}{\tilde{D}} e^{-2D_r t} - \left(\frac{2\alpha^2}{\tilde{D}} t + \frac{6k_B T}{k} + \frac{2\alpha^2}{\tilde{D}} \right) e^{-Dkt/(k_B T)},$$

$$\tilde{D} = \frac{-2D_r k_B T + Dk}{k_B T}.$$

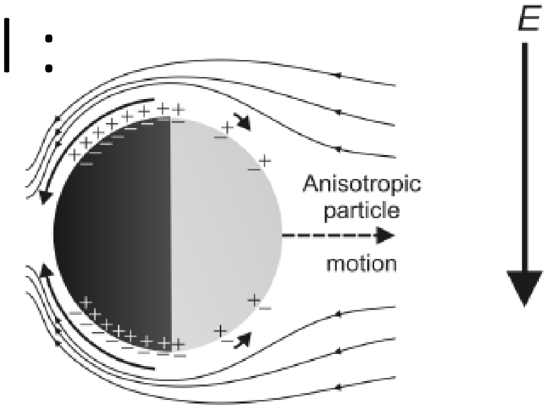
Without potential:

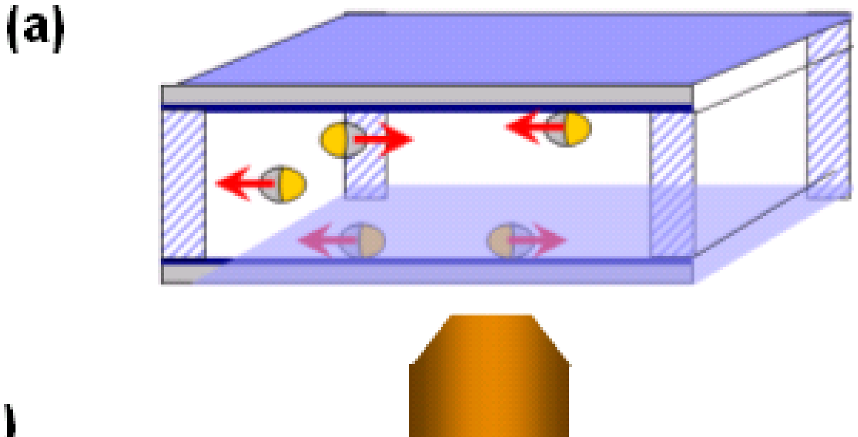
for $k = 0$

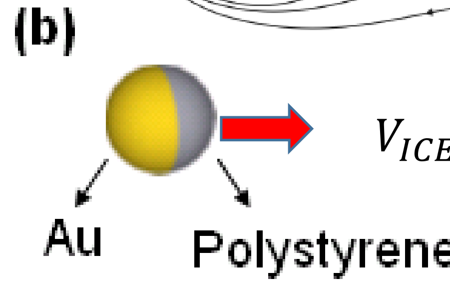
$$< (\mathbf{r}(t) - \mathbf{r}(0))^2 >= 6Dt + \frac{\alpha^2}{2D_r^2} \left(e^{-2D_r t} + 2D_r t - 1 \right),$$

Self-propulsion of Janus particle II :

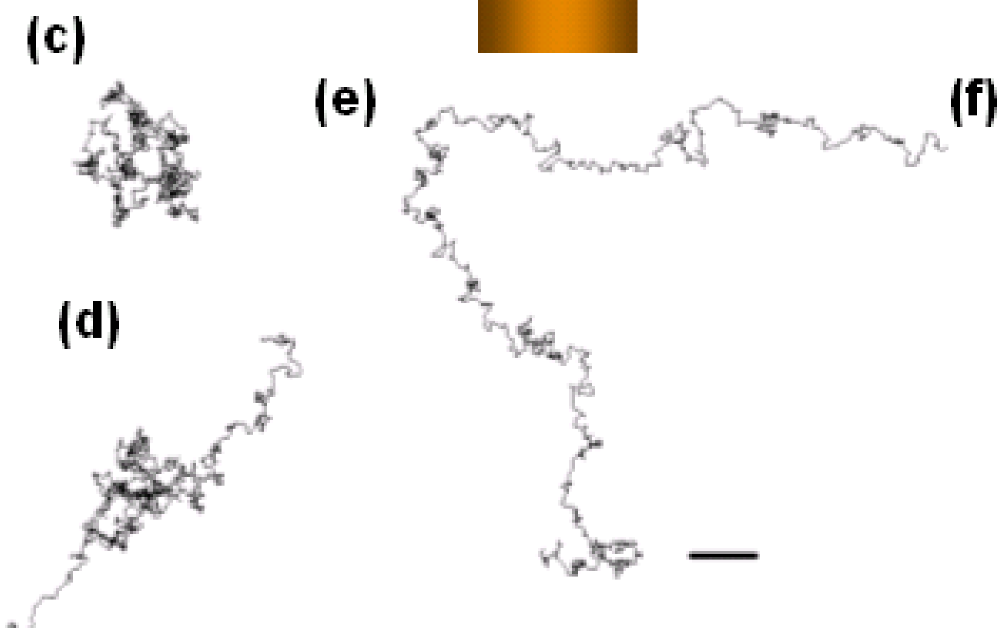
Electric field



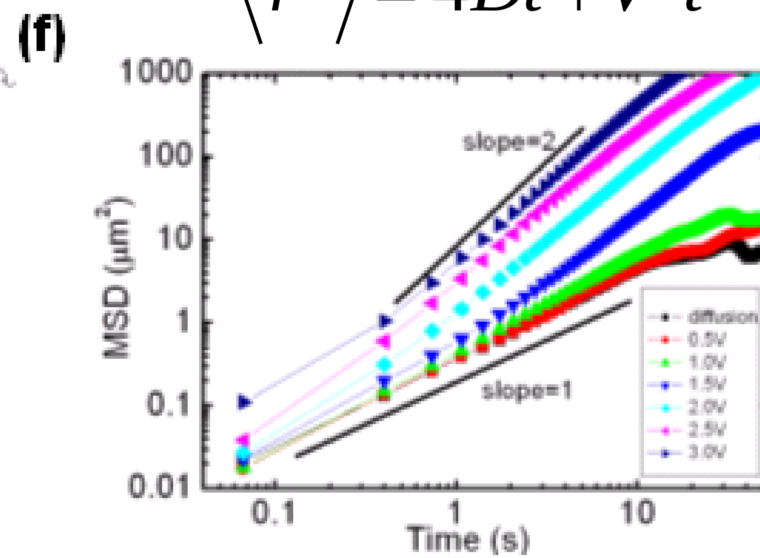




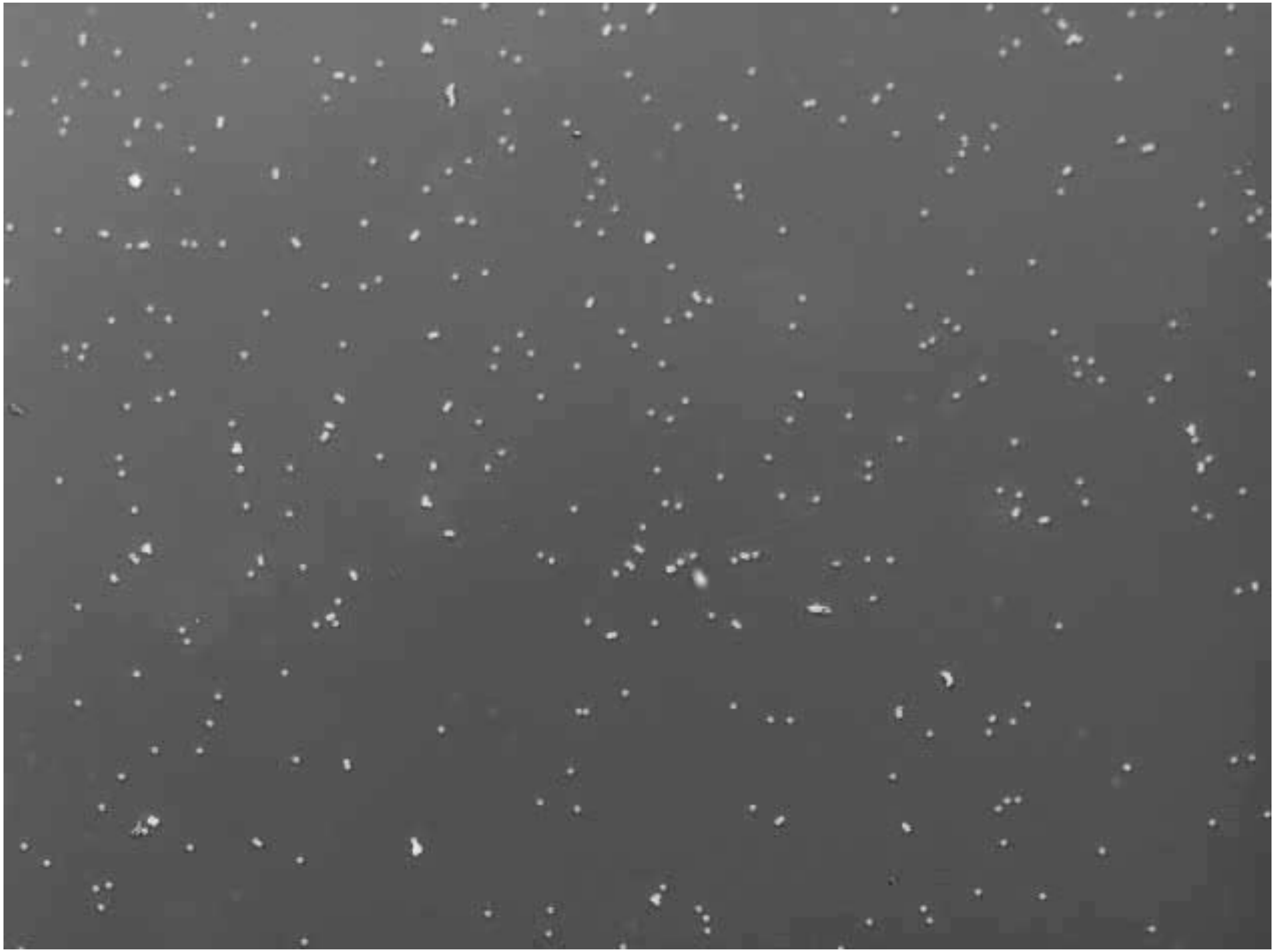
$$V_{ICEO} \sim \frac{\epsilon\varepsilon R E^2}{\gamma}$$



$$\langle r^2 \rangle = 4Dt + V^2 t^2$$



Motion of Janus Particle: Top View



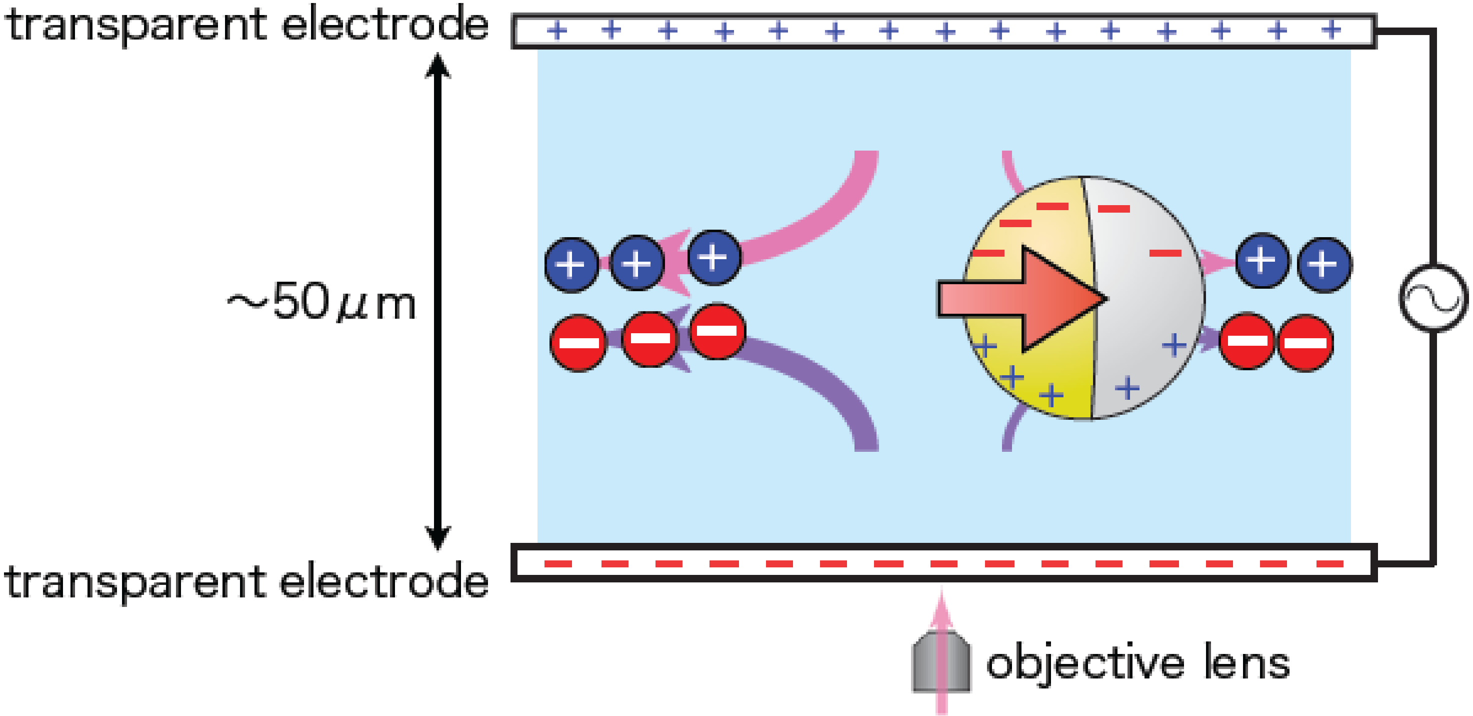
How do they move?

1) Surface charges are induced by the AC electric field $\vec{E}$

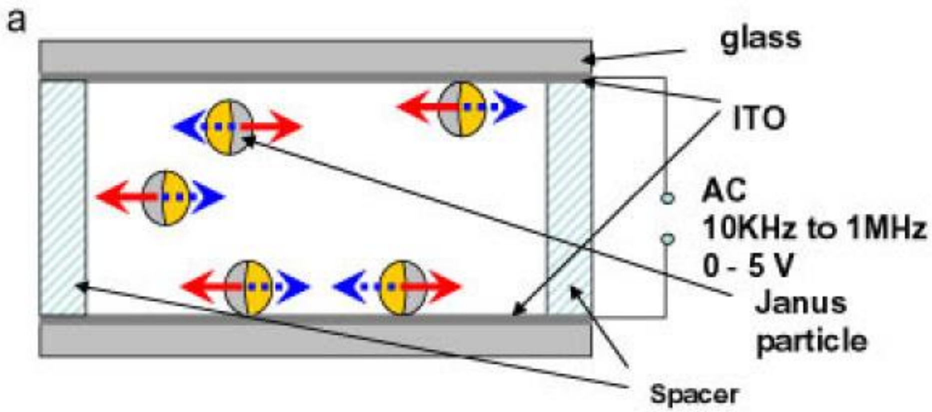
2) Counter-ions gathers around the induced charges.

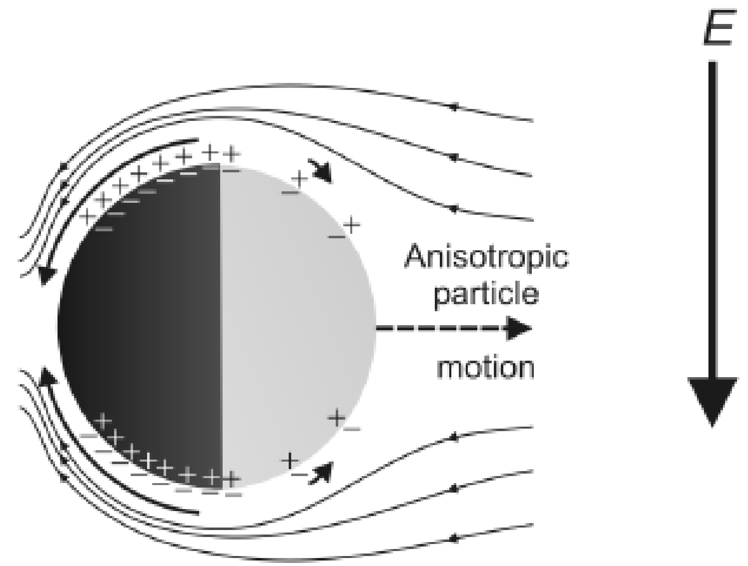
3) Flow of fluid containing counter-ions is induced by $\vec{E}$

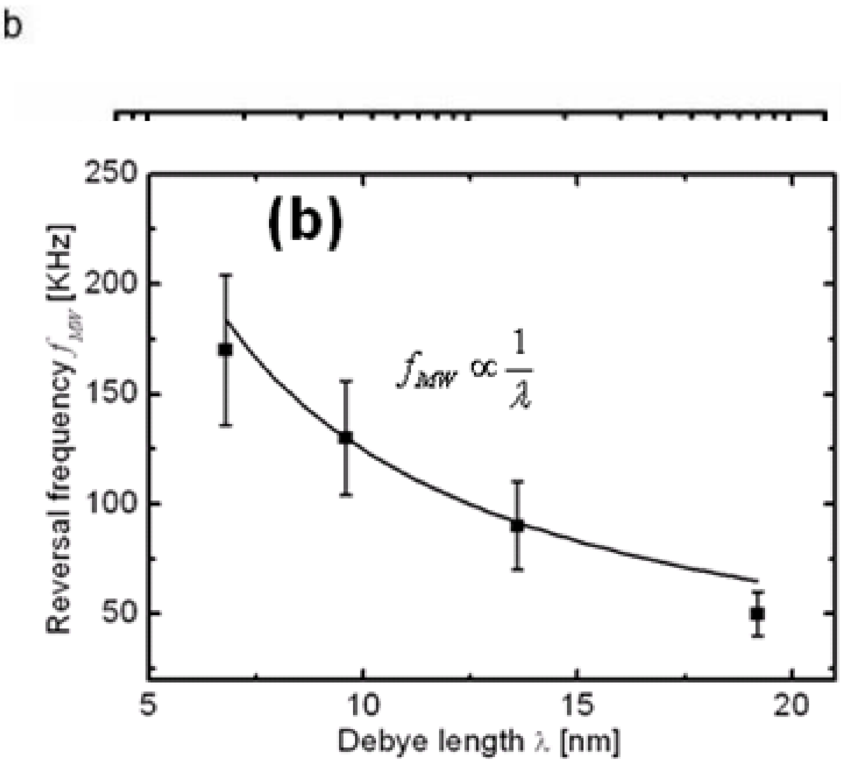
4) Janus particle moves in the opposite direction of the net flow

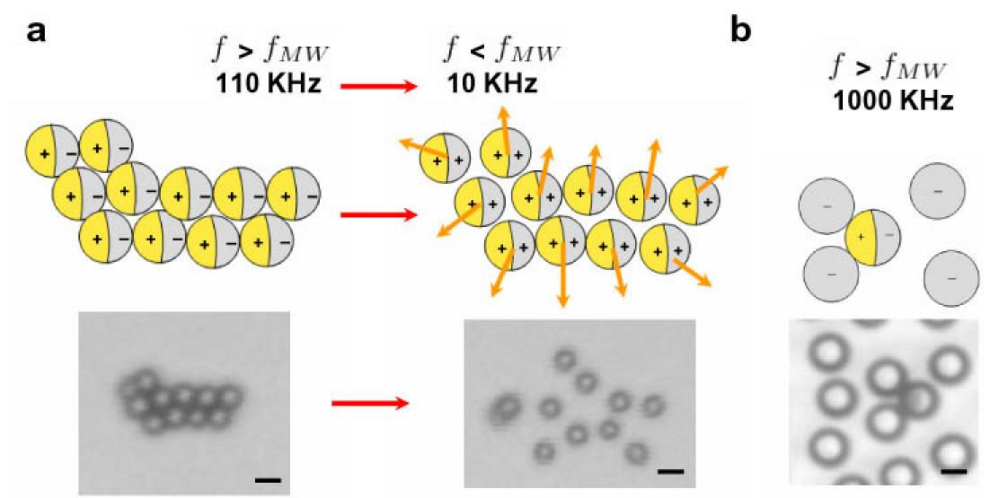


Self-propulsion of Janus particle II :
Electric field

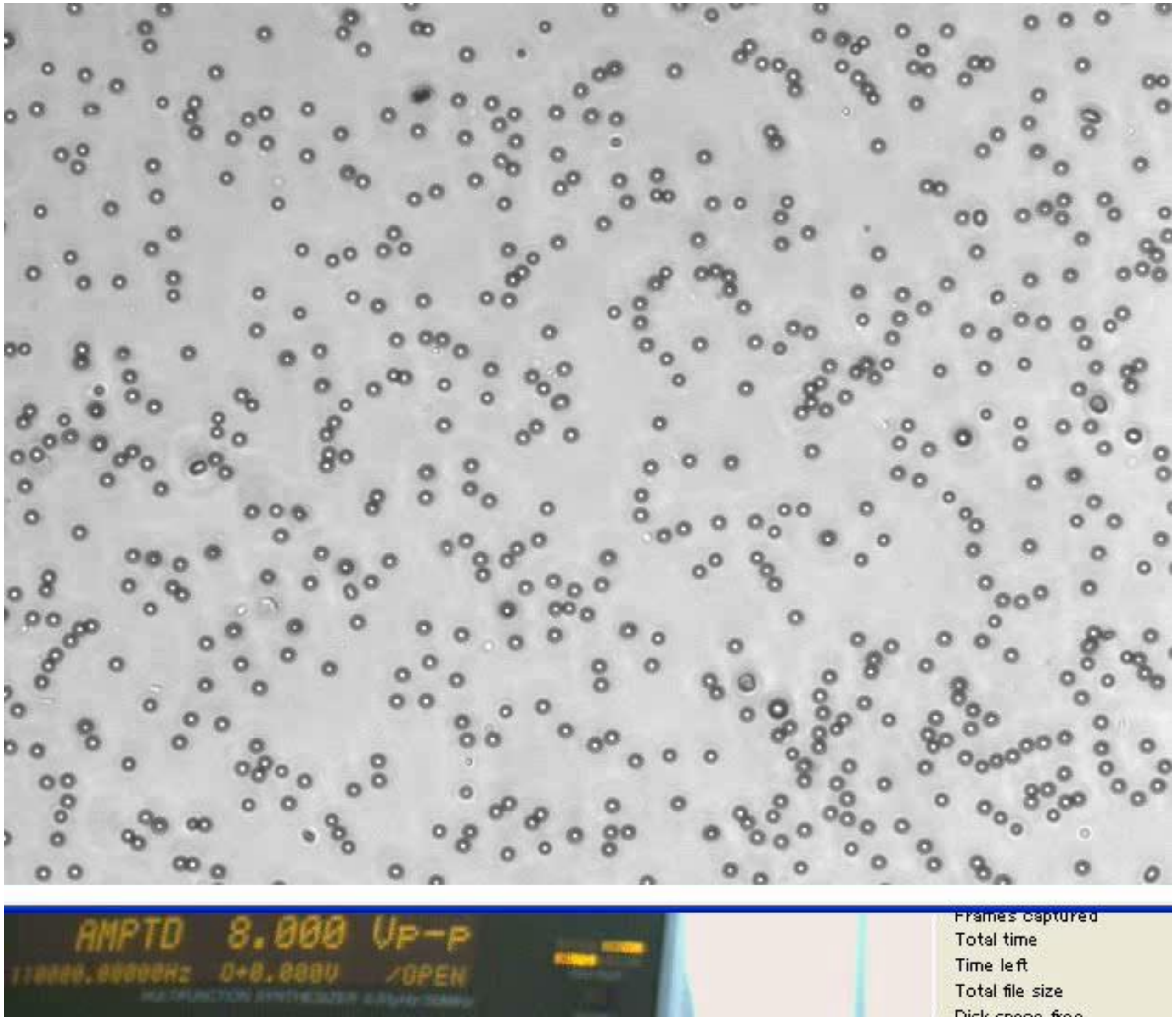


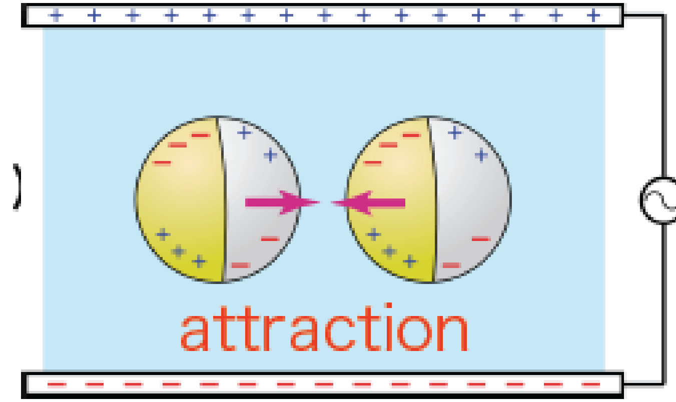


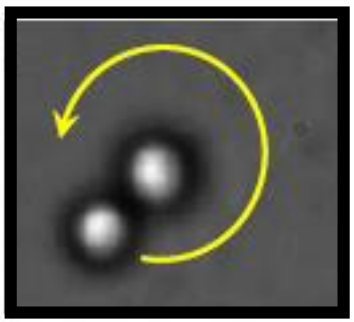




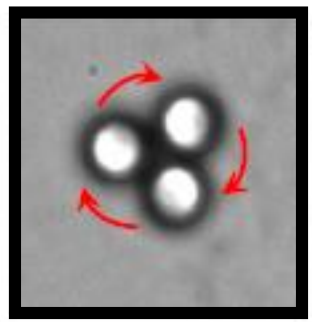
Formation of Chains at frequency higher than fc



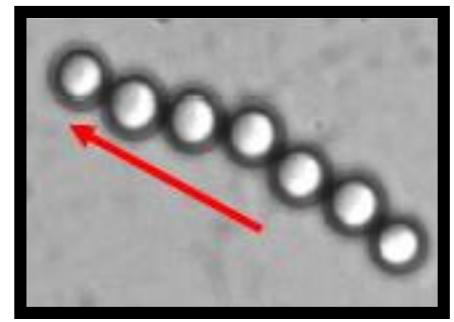




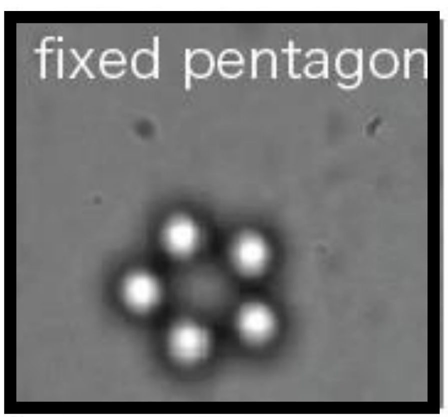
Doublet

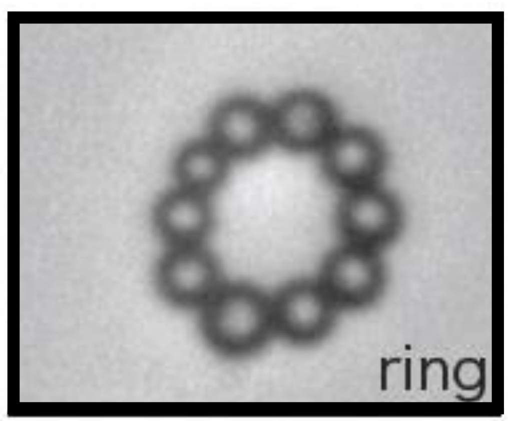


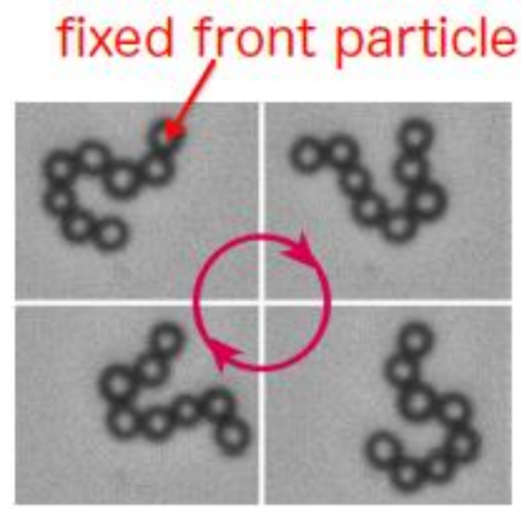
Triplet



Linear Chain

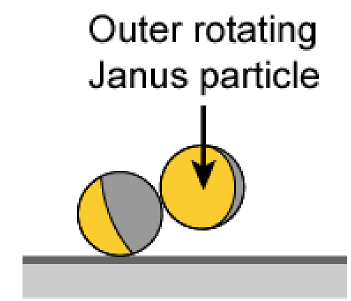






Oscillation, Wave

Fluctuation in Janus Chiral Doublet



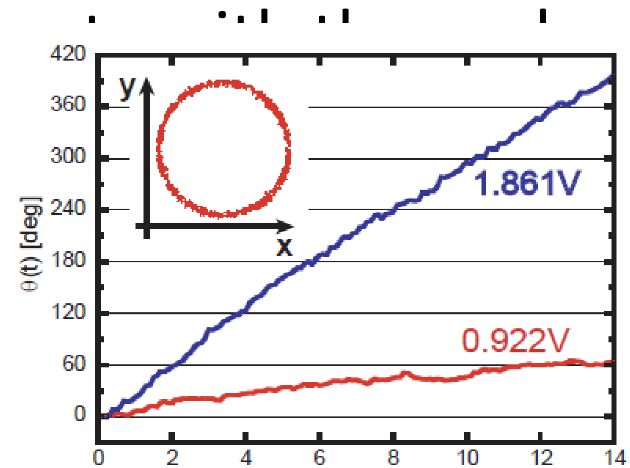
Role of Thermal fluctuation

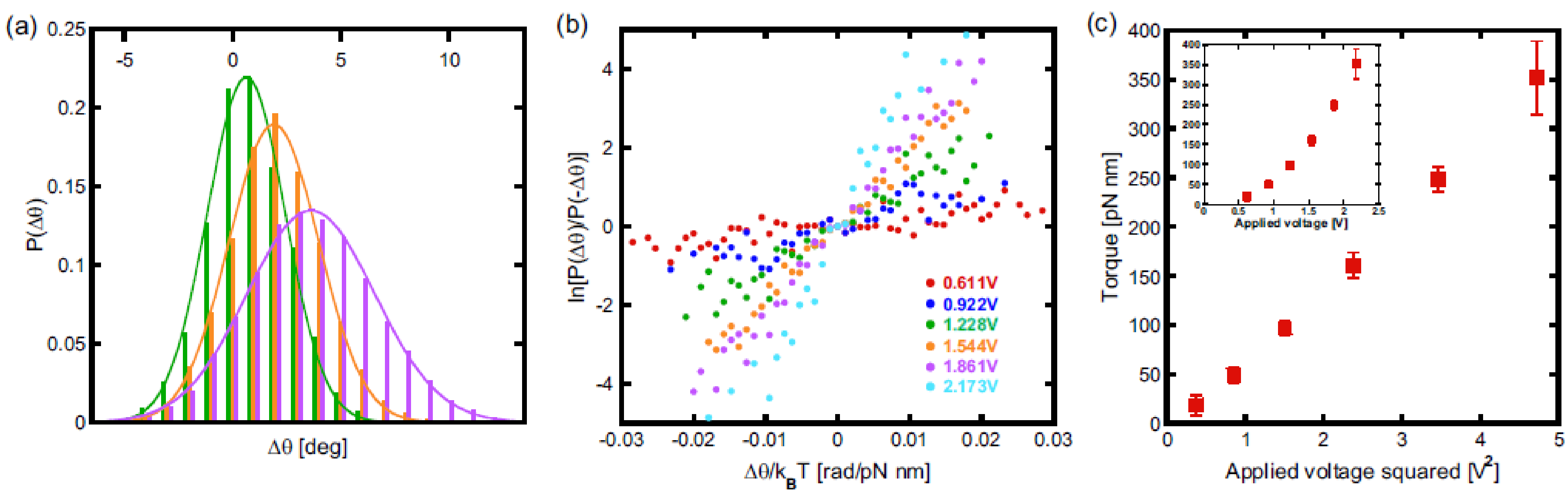
- How does the driving force com
  fluctuations?
- Use of Fluctuation Theorem

$$\ln\left[P(\Delta\theta)/P(-\Delta\theta)\right] = \tau\,\Delta\theta/k_B T$$

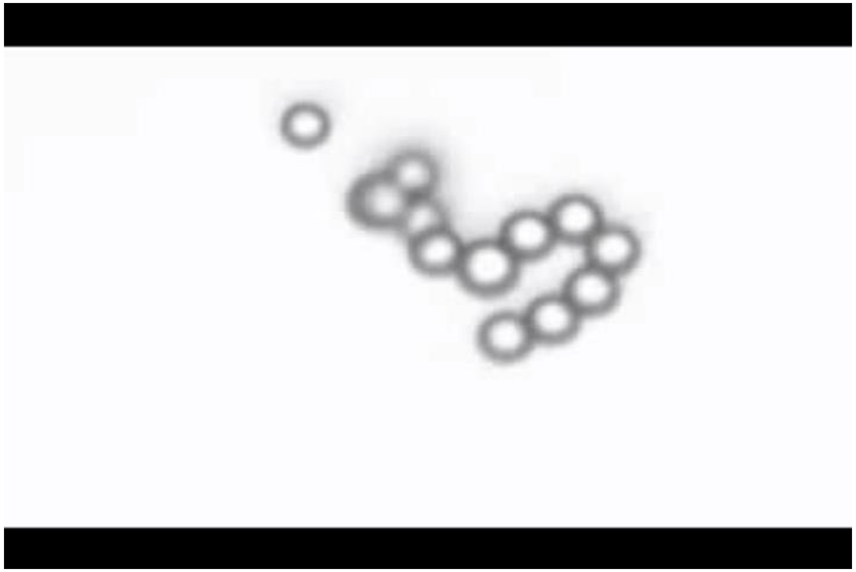
Precise determination of torque is possible

R. Suzuki, HR Jiang, M. Sano, Archive



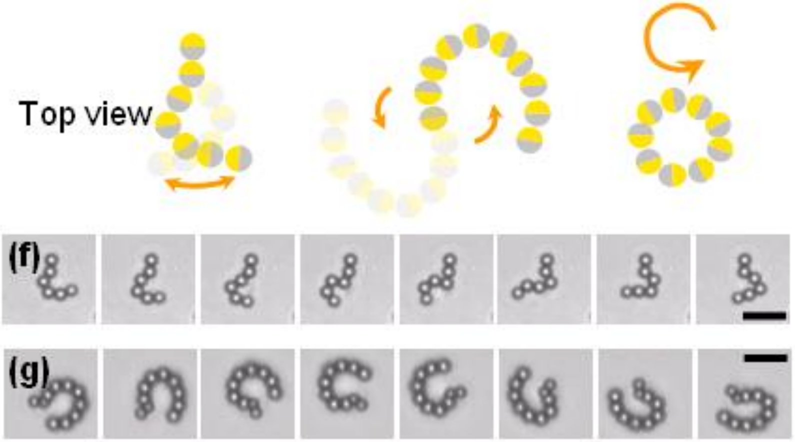


Deformable self-propelled chain



Waving motion

Spiraling motion



Induced Polarization: $P_{eff} = 4\pi a^3 \epsilon_2 Re[K(\omega) E e^{i\omega t}]$

$$K(\omega) = \frac{\epsilon_1 - i\frac{\sigma_1}{\omega} - \epsilon_2 + i\frac{\sigma_2}{\omega}}{\epsilon_1 - i\frac{\sigma_1}{\omega} + 2\epsilon_2 - i\frac{\sigma_2}{\omega}} = \frac{\sigma_1 - \sigma_2}{\sigma_1 + 2\sigma_2}\left(\frac{i\omega\tau_0 + 1}{i\omega\tau_{MW} + 1}\right),$$

$$\tau_0 = \frac{\epsilon_1 - \epsilon_2}{\sigma_1 - \sigma_2}$$
$$\tau_{MW} = \frac{\epsilon_1 + 2\epsilon_2}{\sigma_1 + 2\sigma_2}$$

Electric charging time

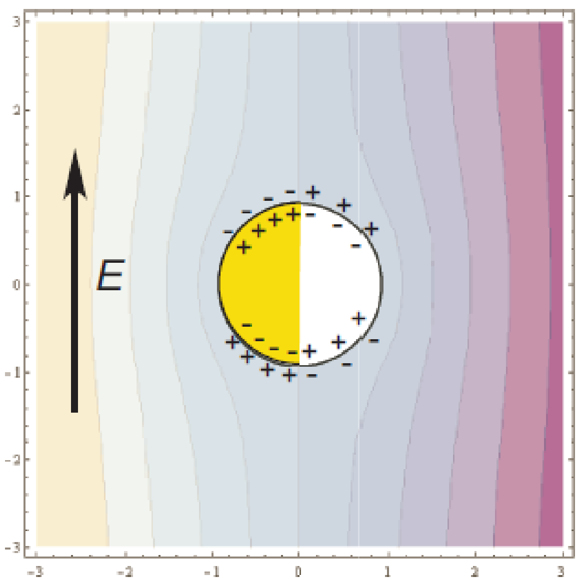


FIG. 5: $f \ll f_c$:

Counter ions in double layer fully charged.

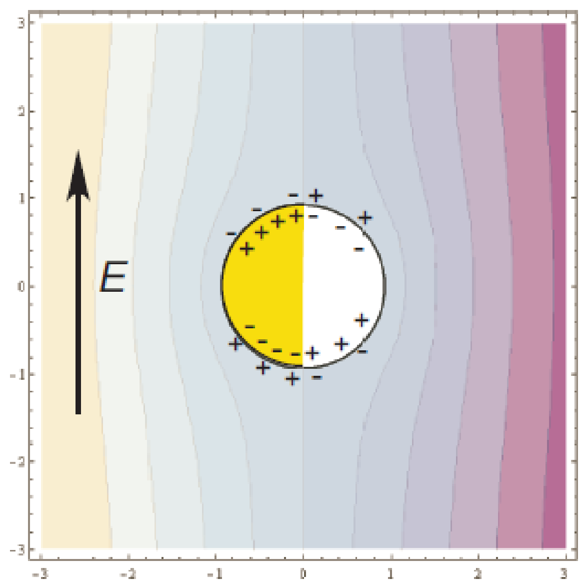


FIG. 6: $f \leq f_c$:

Double layer partially charged.

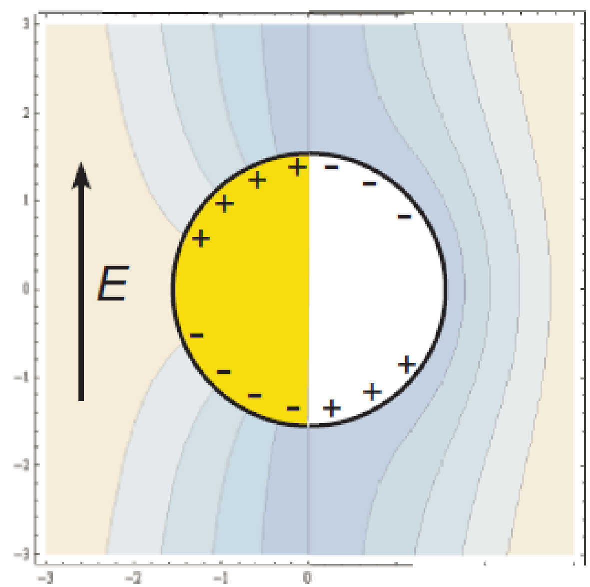
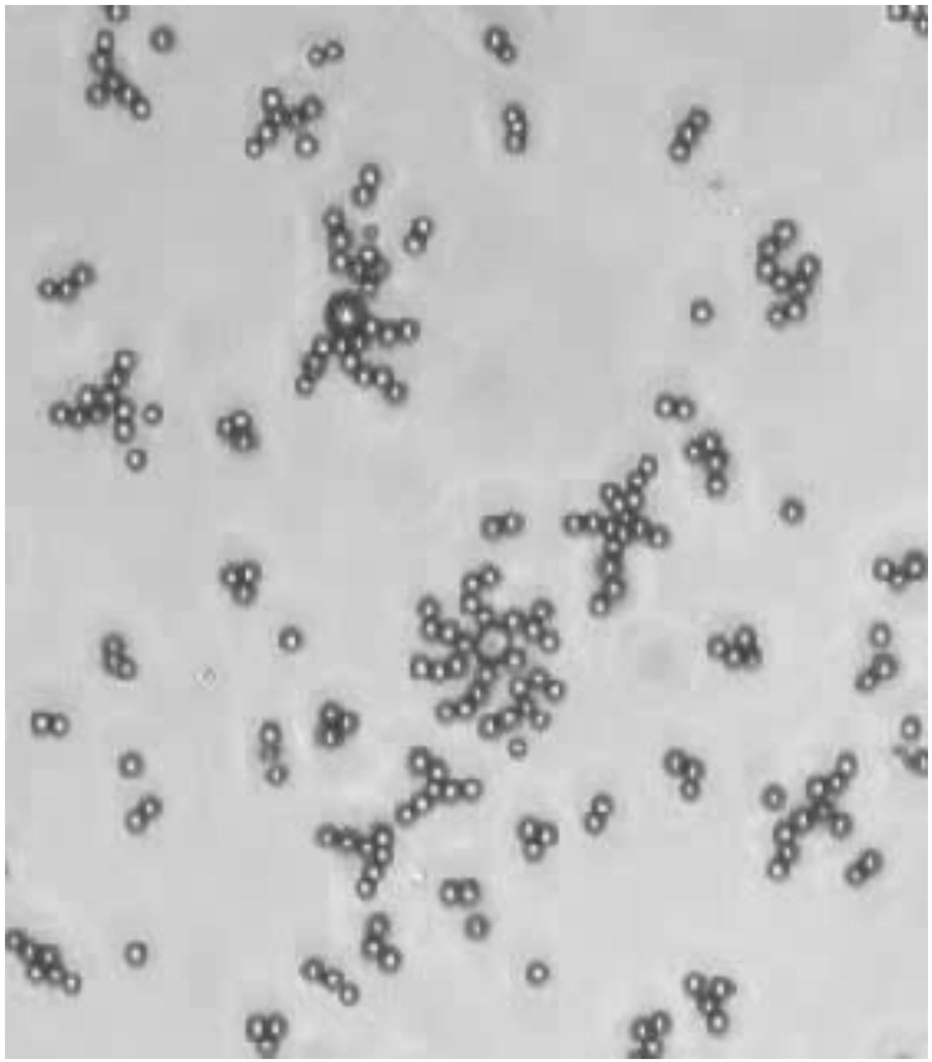


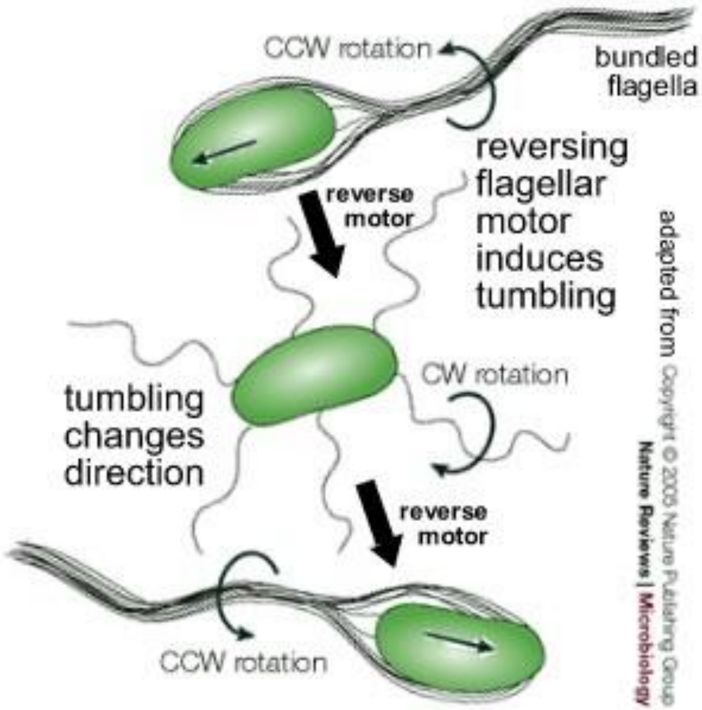
FIG. 7: $f \gg f_c$:

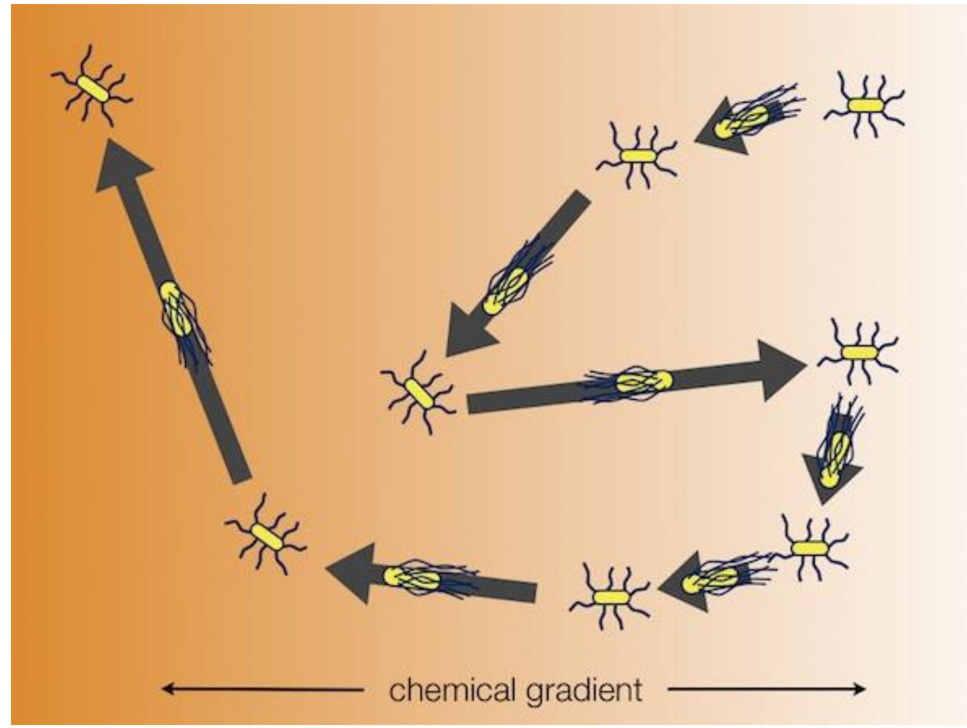
Double layer not charged.

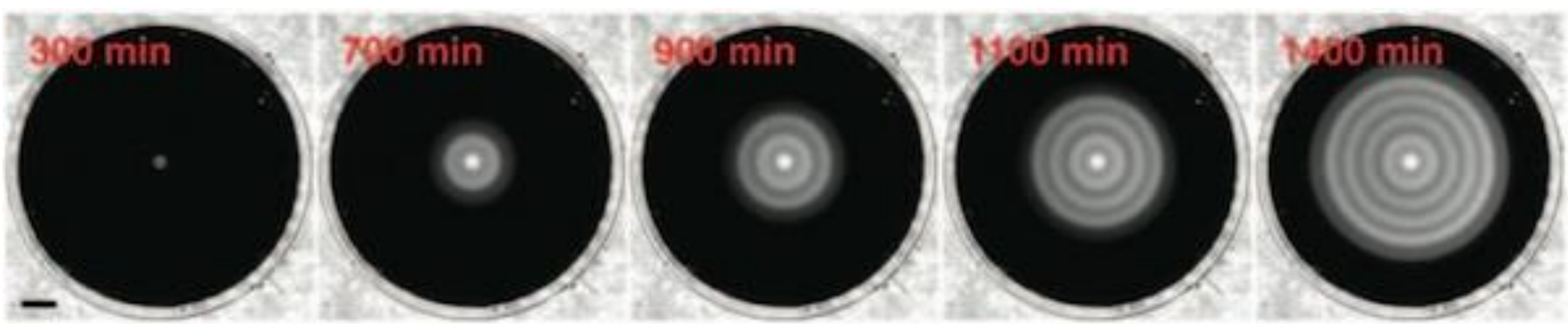
Collective motion of self-propelled string



Chemotaxis of Bacteria







http://www.sciencemag.org/content/334/6053/238.abstrac
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Information and Feedback in Different Systems

	Fluctuation	Information	Feedback	Outcome
Maxwell's demon	Thermal	Speed, position	Biased Choice of fluctuations	Gain Free Energy
Active Particle	Thermal	-------	--------	Enhanced Diffusion
Bacteria (*Escherichia coli*)	tumbling	Chemotactic Signal	Change tumbling freq.	Chemotaxis
Amoeboid cell (*Dictyostelium Discoideum*)	Instability of cell shape	Chemotactic Signal	Biased Choice of random protrusion	Chemotaxis

Summary of Part II

- Information thermodynamics can be tested and demonstrated in colloidal systems.

- Different kinds of phoresis can be used to create self-propelled particles and control interaction of particles.