An Approach to the Analysis of Wrinkling through Scaling Laws and Bounds

(Keynote Talk #3)

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Thin sheets exhibit a daunting array of patterns. A key difficulty in their analysis is that while we have many examples, we have no classification of the possible "patterns."

Recently I have explored a variational perspective on wrinkling, in collaborations with Peter Bella, Jacob Bedrossian, Jeremy Brandman, and Hoai-Minh Nguyen. We focus on the *scaling law* of the minimum elastic energy (with respect to the sheet thickness, and the other parameters of the problem). To identify the scaling law one must prove an upper bound and a lower bound that scale the same way. The upper bound requires a good ansatz, and nature often gives us a hint. The lower bound is typically more subtle, since it must be ansatz-independent. In many cases, the proof of the lower bound helps explain "why" we see particular patterns.

This viewpoint has been implemented in several examples of tension-induced wrinkling, including the wrinkling of an annular sheet loaded in tension on both boundaries [1]. It has also been implemented in some examples of compression-induced wrinkling, including the herringbone pattern often seen in a compressed thin film bonded to a compliant substrate [2]. Other, related work with a similar viewpoint includes [3-5].

My talk will provide an introduction to these developments, and more generally to the advantages and limitations of this approach to wrinkling.

References:

[1] P. Bella and R.V. Kohn, *Wrinkles as the result of compressive stresses in an annular thin film*, Comm. Pure Appl. Math., in press (published online 1 July 2013, DOI: 10.1002/cpa.21471).
[2] R.V. Kohn and H.-M. Nguyen, *Analysis of a compressed thin film bonded to a compliant substrate: the energy scaling law*, J. Nonlin. Sci. 23, 343-362 (2013).
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[4] J. Bedrossian and R.V. Kohn, *Blister patterns and energy minimization in compressed thin films on compliant substrates*, Comm. Pure Appl. Math., in press (available as arXiv:1304.0284).
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