Isostatic Lattice: From Jamming to Topological Surface Phonons

(Keynote Talk #1)

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Frames consisting of nodes connected pairwise by rigid rods or central-force springs, possibly with preferred relative angles controlled by bending forces, are useful models for systems as diverse as architectural structures, crystalline and amorphous solids, sphere packings and granular matter, networks of semi-flexible polymers, and protein structure. The rigidity of these networks depends on the average coordination number z of the nodes: If z is small enough, the frames have internal zero-frequency modes, and they are "floppy"; if z is large enough, they have no internal zero modes and they are rigid. The critical point separating these two regimes occurs at a rigidity threshold, which corresponds closely to what is often referred to as the isostatic point, that for central forces in d-dimensions occurs at coordination number $z_c = 2d$. At and near the rigidity threshold, elastic frames exhibit unique and interesting properties, including extreme sensitivity to boundary conditions, power-law scaling of elastic moduli with (z- z_c), and diverging length and time scales.

This talk will explore elastic and mechanical properties and mode structures of model periodic lattices, such as the square and kagome lattices with central-force springs, that are just on verge of mechanical instability. It will discuss the origin and nature of zero modes of these structures under both periodic (PBC) and free boundary conditions (FBC), and it will derive general conditions [1] (a) under which the zero modes under the two boundary conditions are essentially identical and (b) under which phonon modes are gapped with no zero modes in the periodic spectrum but include zero-frequency surface Rayleigh waves in the free spectrum. In the former situation, lattices are generally in a type of critical state that admits states of self-stress in which there can be tension in bars with zero force on any node, and distortions away from that state give rise to surface modes under free boundary conditions whose degree of penetration into the bulk diverges at the critical state. The gapped states have a topological characterization, similar to that of topological insulators, that define the nature of zero-modes at the boundary between systems with different topology.

Figure 1. Optional. (a) Please ensure the inserted figure(s) have sufficient resolution for printing purpose. (b) 12pt, Justified, CALIBRI.

References:

[1] K. Sun, A. Souslov, X. M. Mao, and T.C. Lubensky, PNAS **109**, 12369-12374 (2012).
[2] C.L. Kane and T.C. Lubensky, Nature Physics **xx**, xxx (2013)