Broken Time-Reversal Symmetry in Periodic Resonator Arrays

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Time reversal symmetry (TRS) in Macroscopic Maxwell’s Equation

\[ \nabla \times E = i \omega \mu \cdot H \]
\[ \nabla \times H = -i \omega \varepsilon \cdot E \]

\[ t \rightarrow -t \]

\[ \nabla \times E^* = -i \omega \mu \cdot H^* \]
\[ \nabla \times H^* = i \omega \varepsilon \cdot E^* \]

If the Maxwell’s equations are still satisfied after time reversing everything except the materials, we say that the system (material) has TRS

Condition of TRS system
For Macroscopic Maxwell’s equations:

\[ \begin{align*}
\varepsilon^* &= \varepsilon \\
\mu^* &= \mu
\end{align*} \]
Consequence of Time Reversal Symmetry on Band Structures

If $E^*(x)e^{i(kx-\omega t)}$ is a solution,

$E(x)e^{i(-k^*x-\omega t)}$ is also a solution.

Symmetry in band structure

$\omega(k^*) = \omega(-k)$

For pass band with real $k$, we have $\omega(k) = \omega(-k)$

Even there is no spatial symmetry other than periodicity
Broken TRS by Static Magnetic Field

Gyromagnetic materials
\[
\mathbf{\mu} = \begin{pmatrix}
\mu & i\kappa_m & 0 \\
-i\kappa_m & \mu & 0 \\
0 & 0 & \mu_3 \\
\end{pmatrix}
\]

\[
\mathbf{\varepsilon} = \begin{pmatrix}
\varepsilon & i\kappa_e & 0 \\
-i\kappa_e & \varepsilon & 0 \\
0 & 0 & \varepsilon_3 \\
\end{pmatrix}
\]

Such as ferromagnetic materials and plasma under external static magnetic field

Asymmetry in band structure

This will give \( \omega(k) \neq \omega(-k) \) in some situation.
Broken Reciprocity

Non-reciprocal Medium

$$\varepsilon^T \neq \varepsilon$$
$$\mu^T \neq \mu$$

$$\tilde{G}(x_1, x_2) \neq \tilde{G}^T(x_2, x_1)$$
for some positions

Geometrical Requirement:
$$\varepsilon(x) \neq R(\varepsilon(x))$$ for any possible non-identity symmetry operator $R$

It is not the same as broken TRS
Example to break the reciprocity

Decoupling due to splitting of photon angular momentum states

\[
\begin{align*}
\mu_r &= 1 + \frac{\omega_m \omega_n}{\omega_n^2 - \omega^2} \\
\mu_r &= \frac{\omega_m \omega}{\omega_n^2 - \omega^2} \\
\omega_n &= \gamma H_0 \\
\omega_m &= \gamma M_0
\end{align*}
\]

Gyromagnetic cylinder

\[
\vec{\mu} = \begin{pmatrix}
\mu_r & -i \mu_k & 0 \\
 i \mu_k & \mu_r & 0 \\
 0 & 0 & 1
\end{pmatrix}
\]

Jin Wang et al., PRB 84, 235122 (2011)
Mode decoupling phenomena - 1

Jin Wang et al., PRB 84, 235122 (2011)
Splitting of photon angular momentum states

- Frequency splitting diagram

Jin Wang et al., PRB 84, 235122 (2011)
Comparing perturbation formula with exact results

Off-diagonal term of inverse permeability

\[ \Delta \omega \propto j_z \cdot \frac{\mu_k}{\mu_k^2 - \mu_r^2} \]

Angular momentum density at surface

Asymmetry in (discrete) band structure for edge state of single resonator

\[ \exp(i m \varphi) \leftrightarrow \exp(i k x) \]

Jin Wang et al., PRB 84, 235122 (2011)
TRS breaking

- Many refer to the breaking of symmetry when exchanging source and sink
  - Not the true meaning of TRS breaking:
    \[ \varepsilon^* \neq \varepsilon \quad \text{or} \quad \mu^* \neq \mu \]

- What if we consider situation where there is always a symmetry between source and sink due to geometry?
- What are the possible consequences of broken TRS in these systems?
Example: Mode decoupling in array of gyromagnetic resonators

Degenerate state

Non-degenerate state

Jin Wang et al., PRB 84, 235122 (2011)
Decoupling among resonators

- **Group velocity**
  - Strong coupling → High group velocity
  - Weak coupling → Low group velocity

- Could offer new ways to achieve dense arrays of independent resonators

- Localizing light with static magnetic field?
Breaking TRS without static magnetic field

Simply by absorption loss: \( \text{Im}(\varepsilon) \neq 0 \)
\[ \rightarrow \varepsilon^* \neq \varepsilon \]

Usually, we have

\[
\begin{align*}
\varepsilon^T & = \varepsilon \\
\mu^T & = \mu
\end{align*}
\]

\[ \mathbf{\hat{G}}(x_1, x_2) = \mathbf{\hat{G}}^T(x_2, x_1) \]

Then, \( \omega(k) = \omega(-k) \)?
Symmetric Band Structure?

Any degeneracy lifted by absorption?
Transverse hybrid modes

In the case of no absorption $\omega(k^*) = \omega(-k)$ must be satisfied because we have TRS.

In pass band $k$ is real, so $\omega(k) = \omega(-k)$

In band gap $k$ is complex, so
\[
\omega(Re(k) - i Im(k)) = \omega(-Re(k) - i Im(k))
\]

Two degenerate modes propagating in the same direction.
Complex-\(k\) band structure

Initial decay length = \(\frac{1}{\text{Im}(\kappa)}\)

\[ \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \]

Answer: The lifting of degeneracy is in the decay length

**Eigen-decomposition**

\[
|P\rangle = \sum_j \lambda_j^{-1} |P_j\rangle \langle P_j| E_{\text{exc}} \rangle
\]

\[
p_n = p_0 \frac{a}{2\pi} \int_{-\pi/a}^{\pi/a} \frac{e^{ika}}{\lambda(\omega, k)} dk
\]

\[
p_n = \frac{p_0}{2\pi i} \oint_{|z|=1} g(\omega, z) z^{n-1} dz \quad (z \equiv e^{ika})
\]

**MNPs**

- Dipole source

**Poles → Exponential Decay**

**Branch cut → Non-exponential Decay**

**KH Fung et al., Opt. Lett. 36, 2206 (2011).**
Radiation vs. Absorption

\[ F(m) = G(\omega, m) \]

1. \[ F(m) = c_1 z_1^m \]
2. \[ F(m) = c_2 z_2^m \]

Summary

- Broken time-reversal symmetry in periodic resonator arrays
- Two effects discussed
  - Decoupling between nearly touching resonators
  - Lifting of degeneracy in complex band structure
- There could be more “hidden” properties that are associated with broken time-reversal symmetry
Thank you!

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